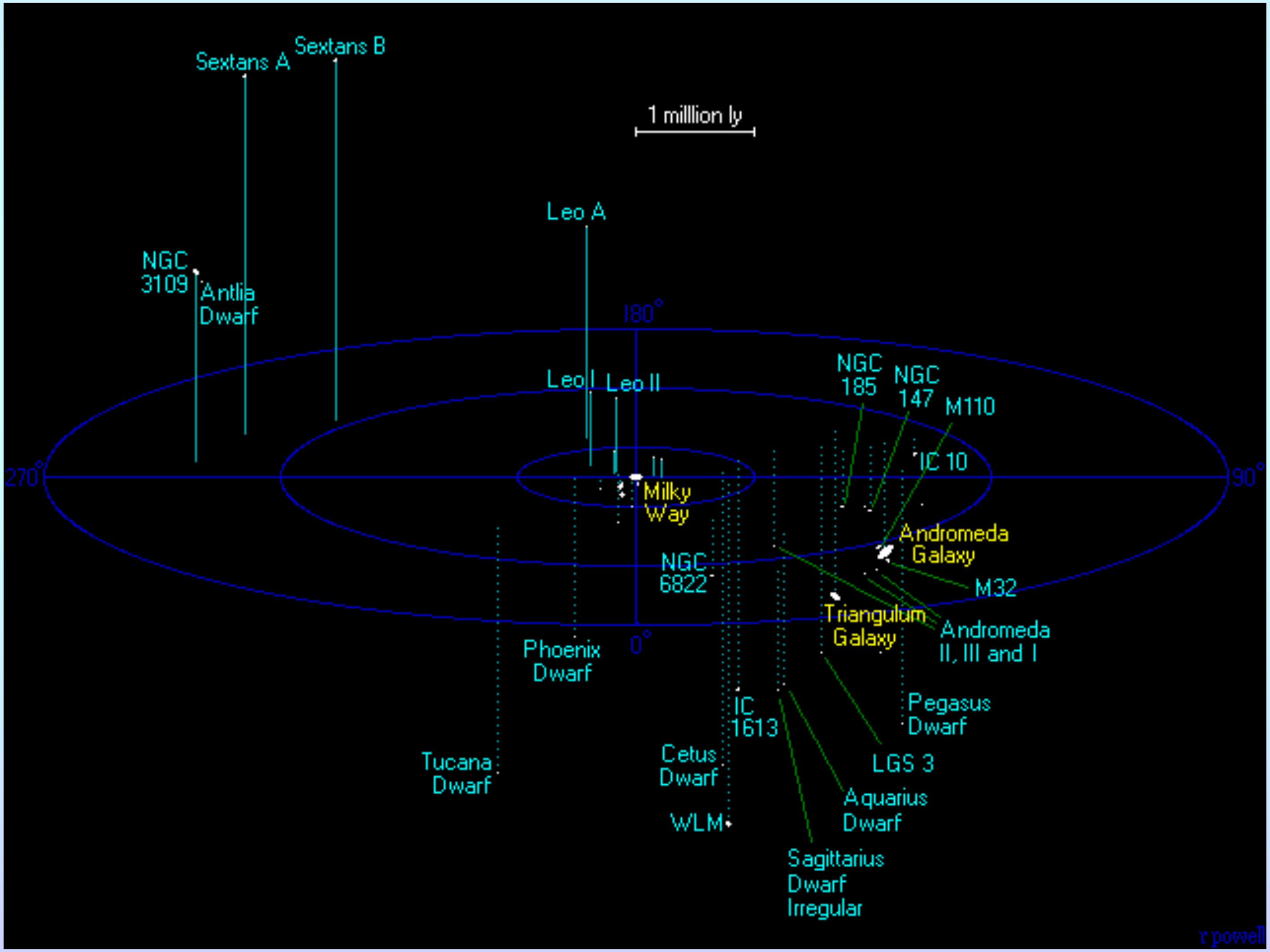
The Local Group

Most galaxies are clumped together in small groups or large galaxy clusters. Our Galaxy is part of the Local Group.



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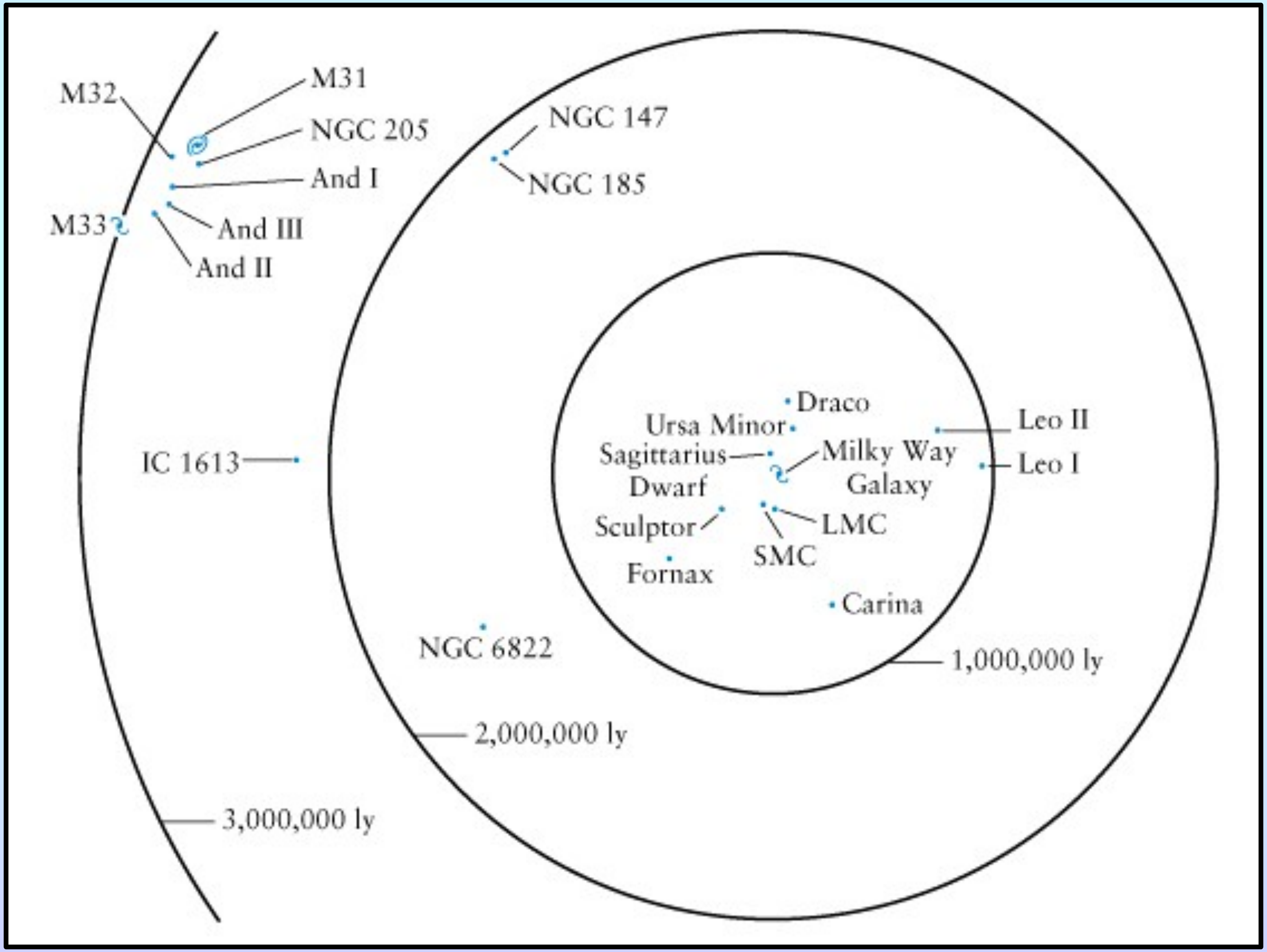
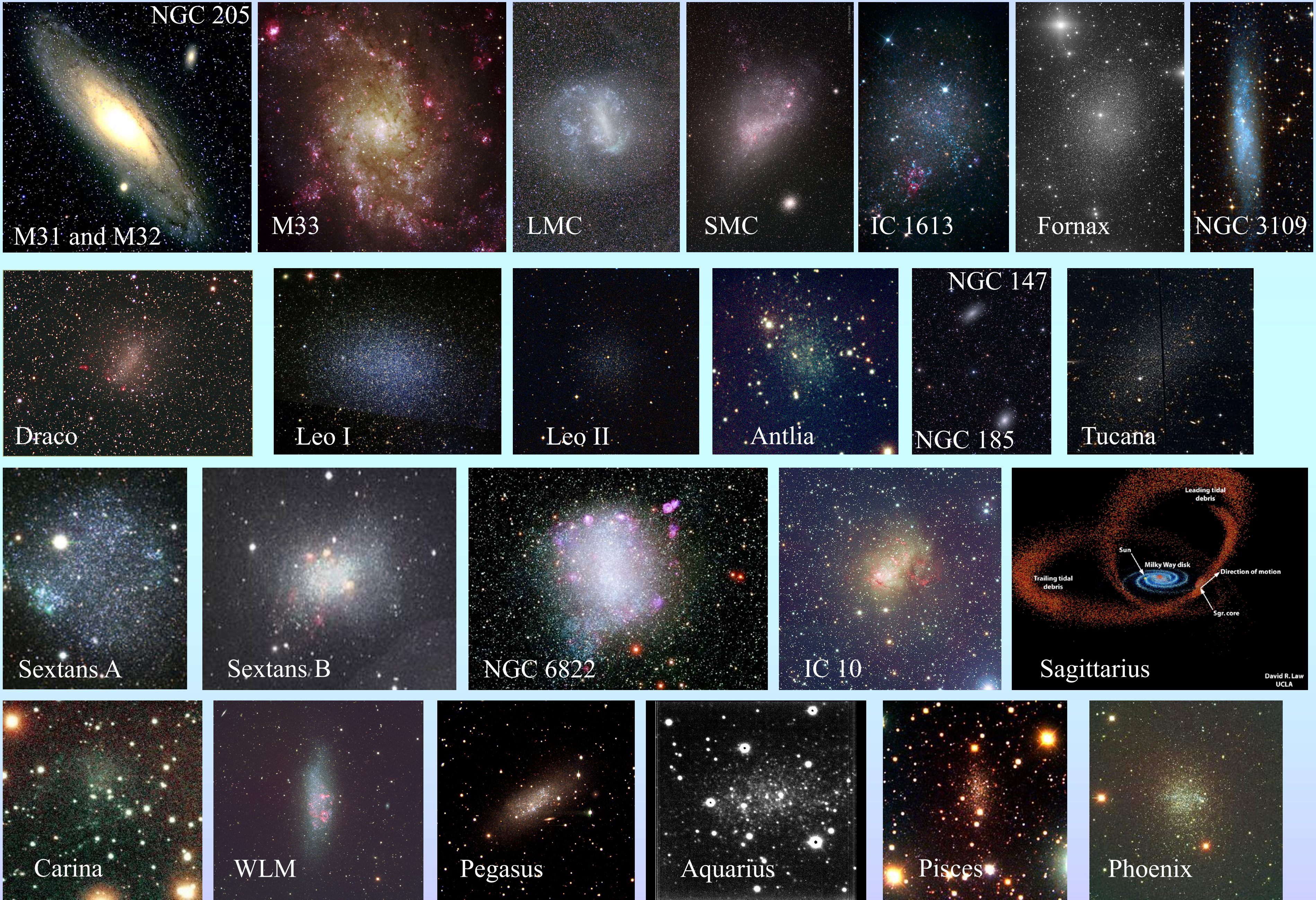
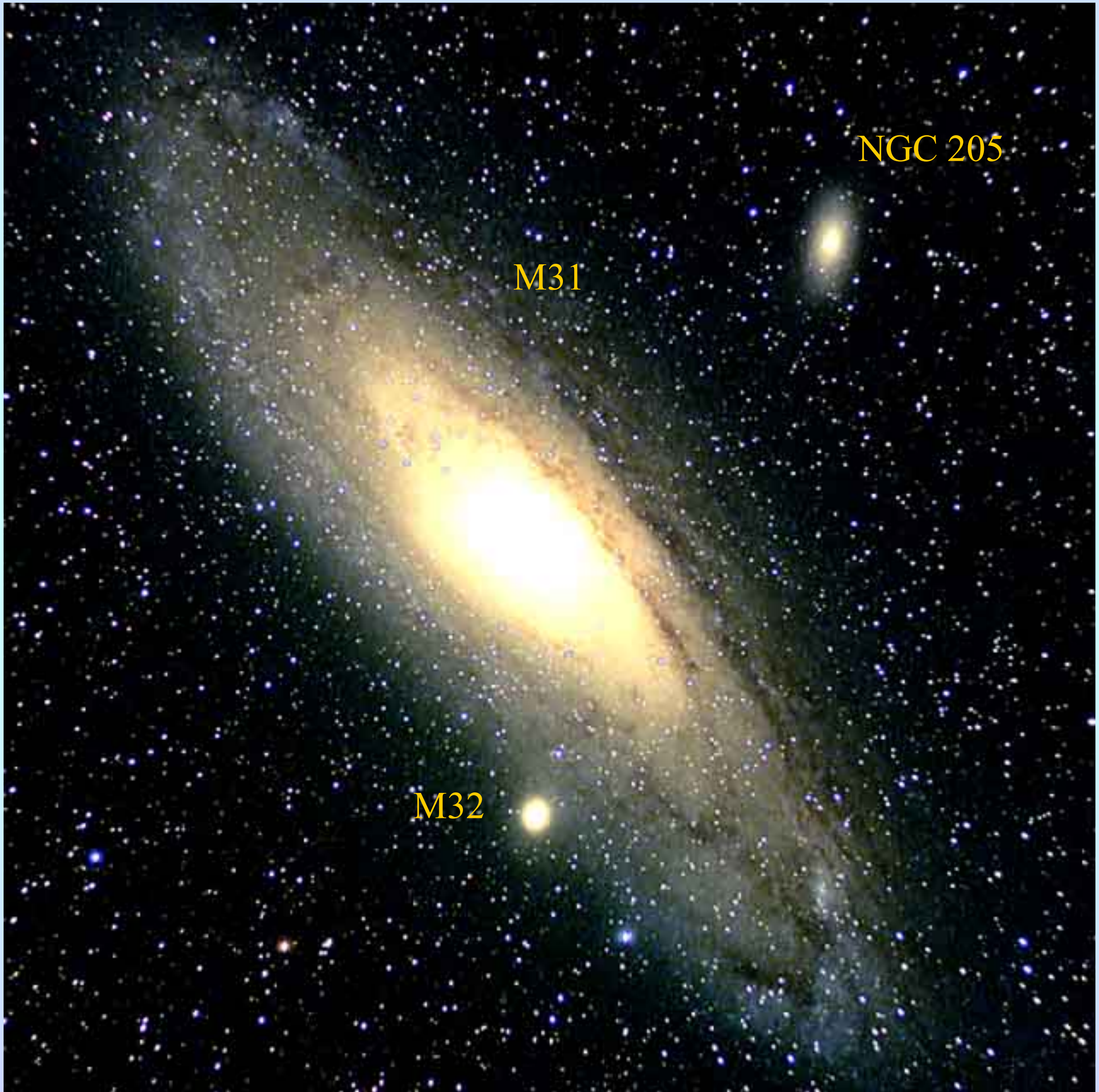
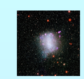


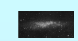
The Local Group



Local Group Galaxies

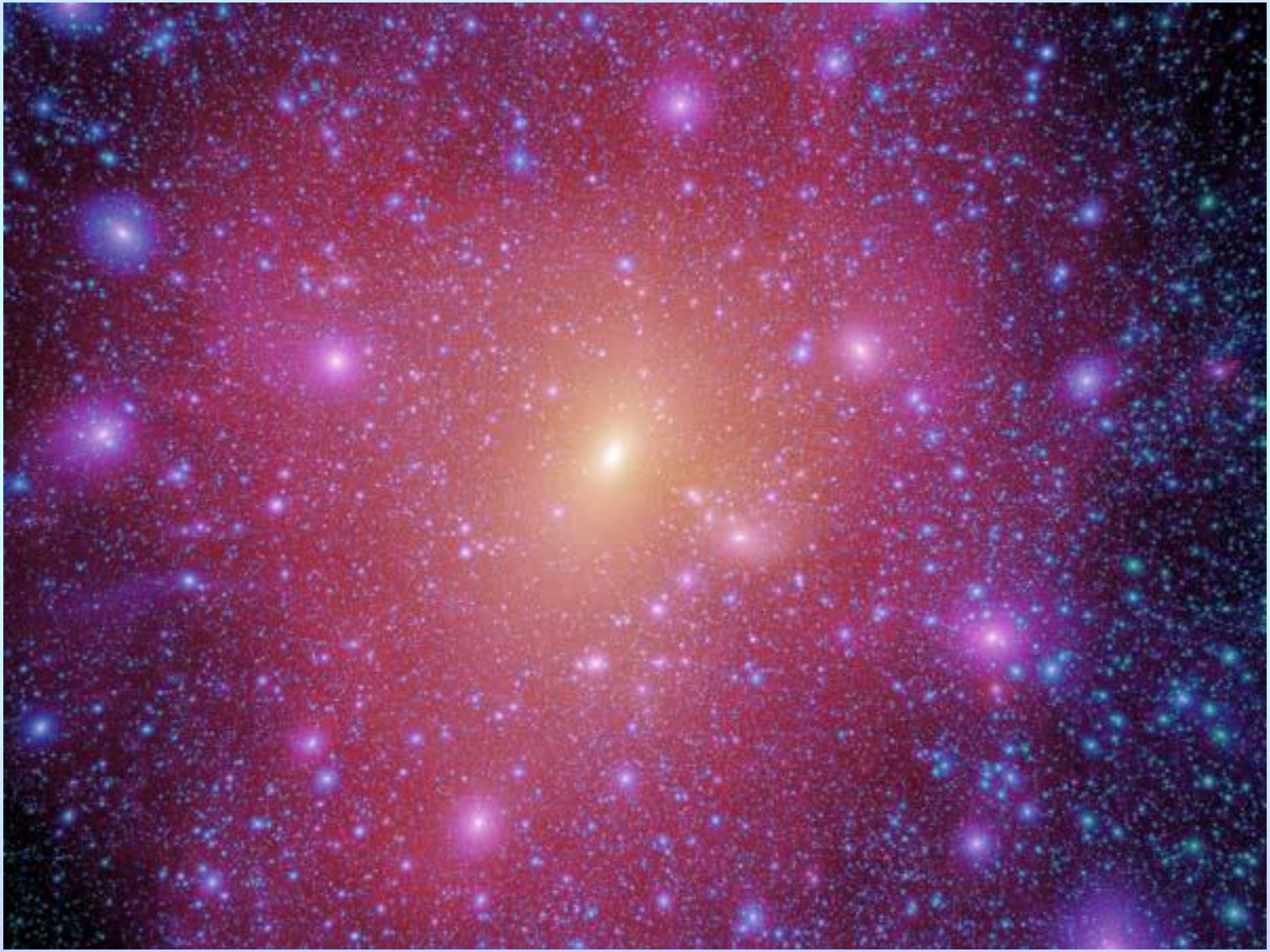




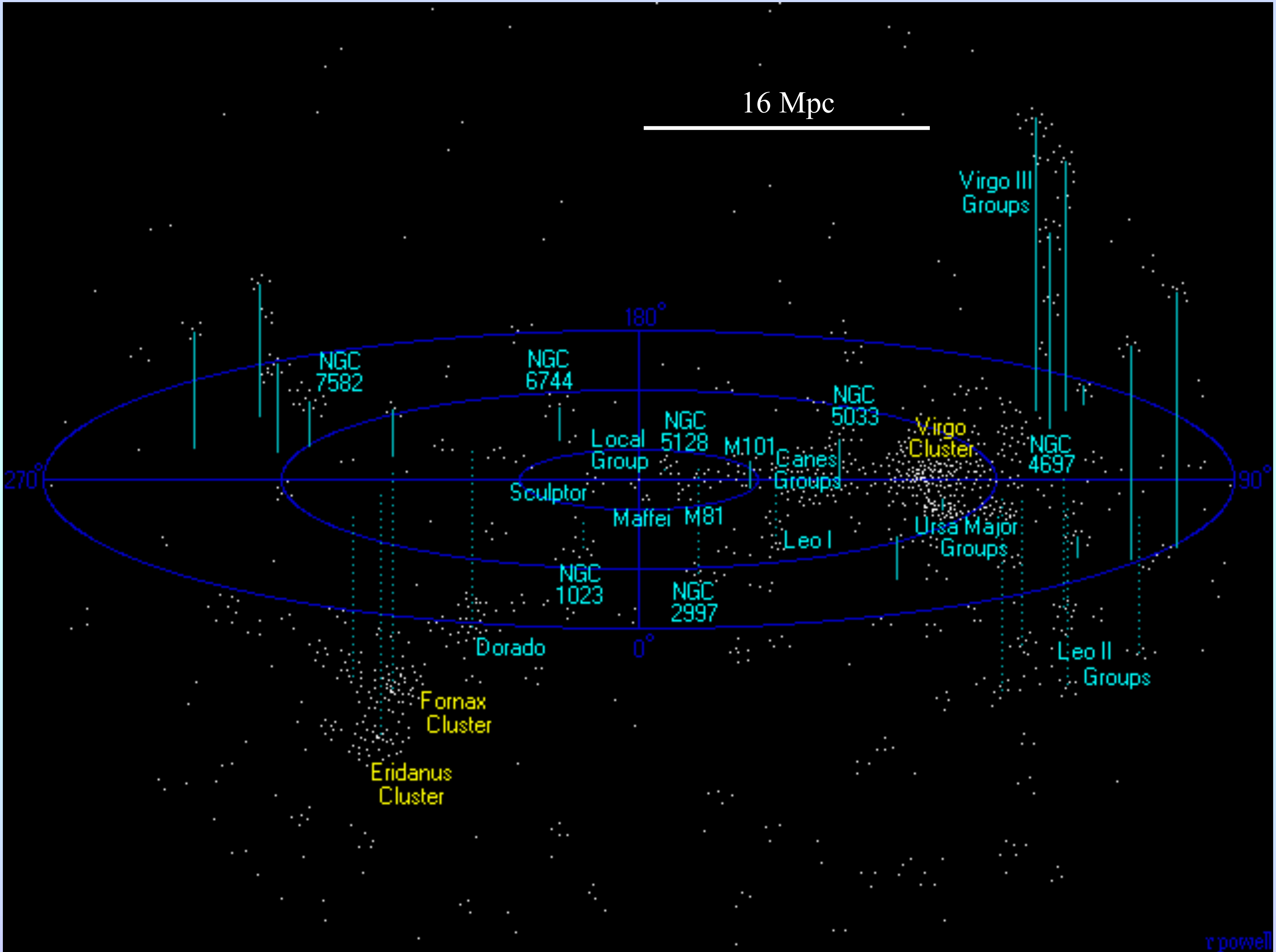


Plus about 50+ other galaxies too small to include … and the Milky Way.

Dark Matter Computer Model of Milky Way and Satellite Dwarf Galaxies



The Local (Virgo) Supercluster



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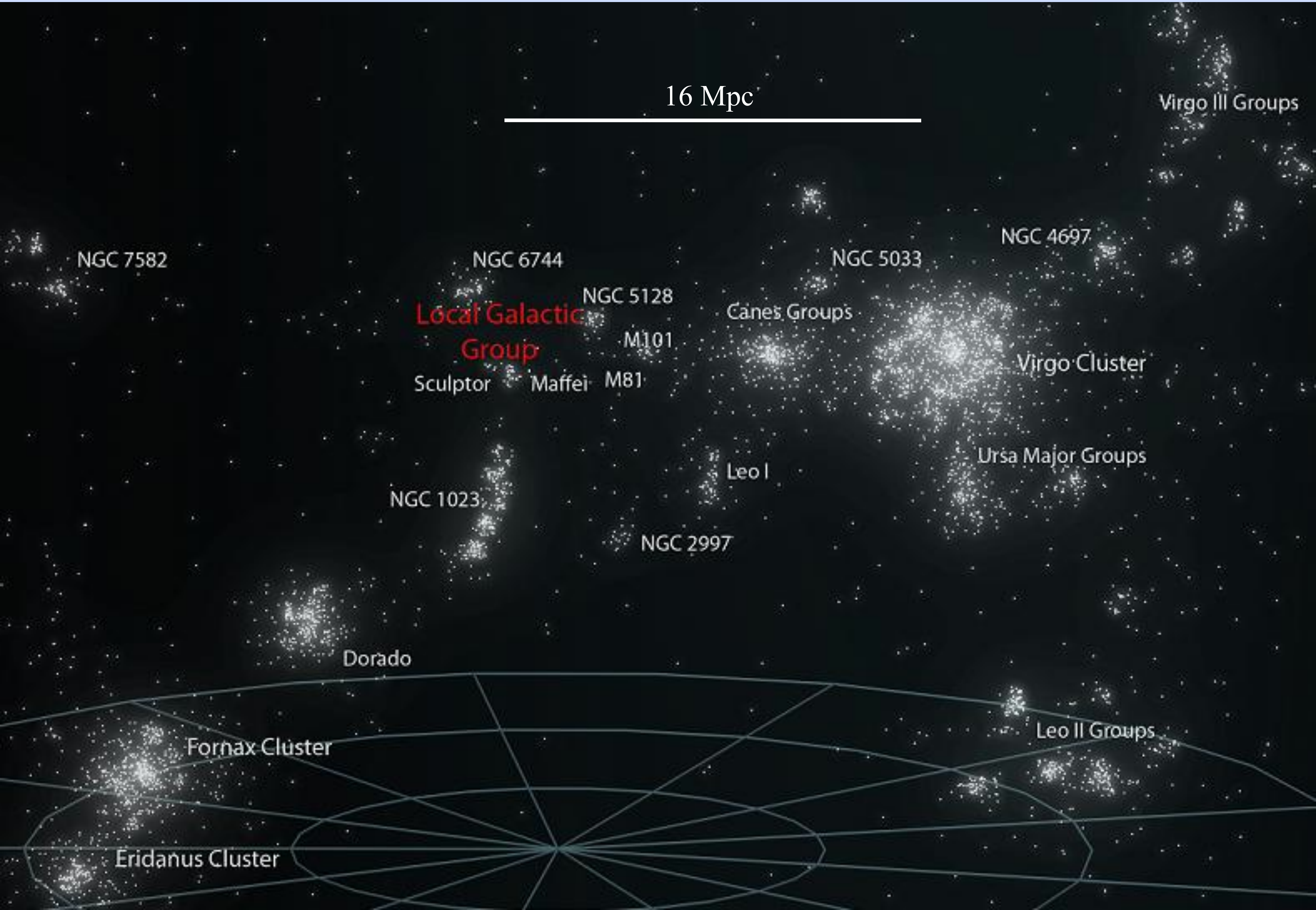
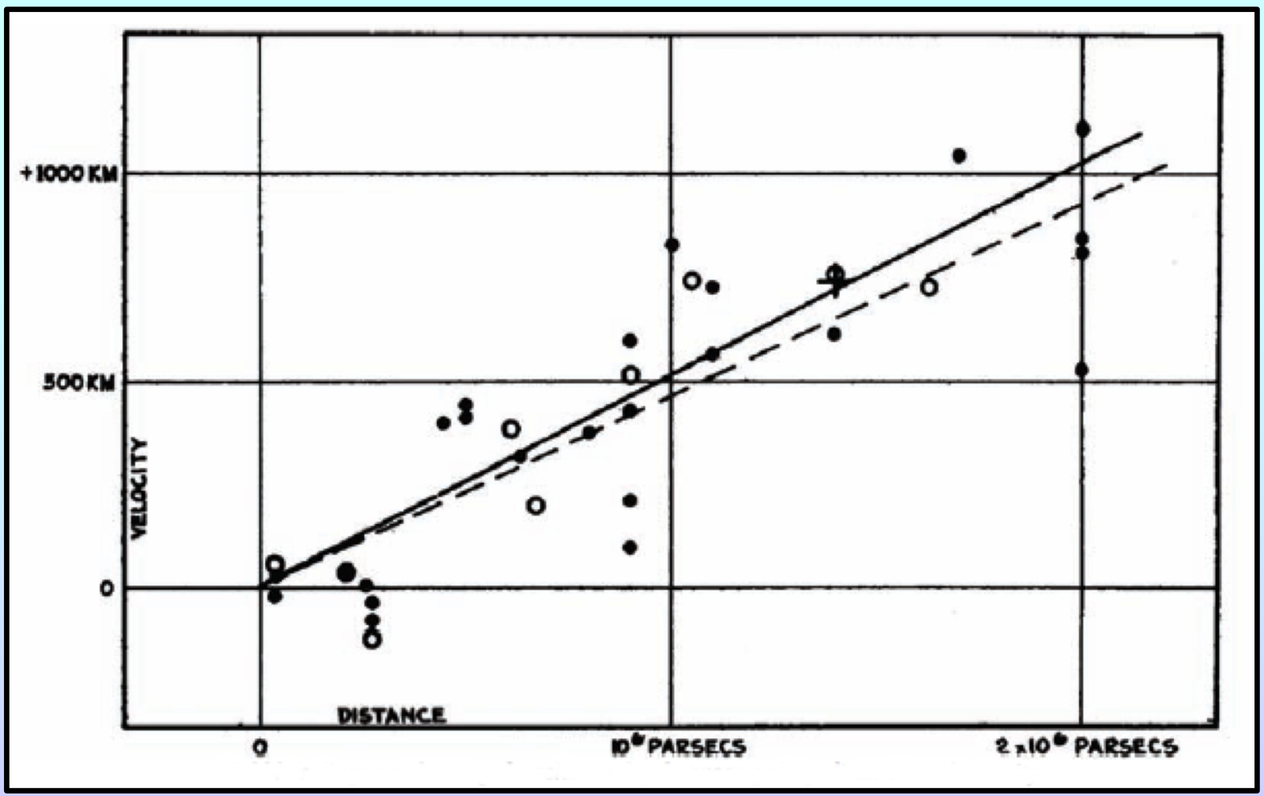


Galaxy Distances

In 1929, Edwin Hubble discovered a linear relation between the approximate distances to galaxies and their recessional velocity. In short, $v = H_0 D$. (Much more on this later.) But to apply this law, one first needs to measure H_0. That means measuring precise distances to calibration galaxies.



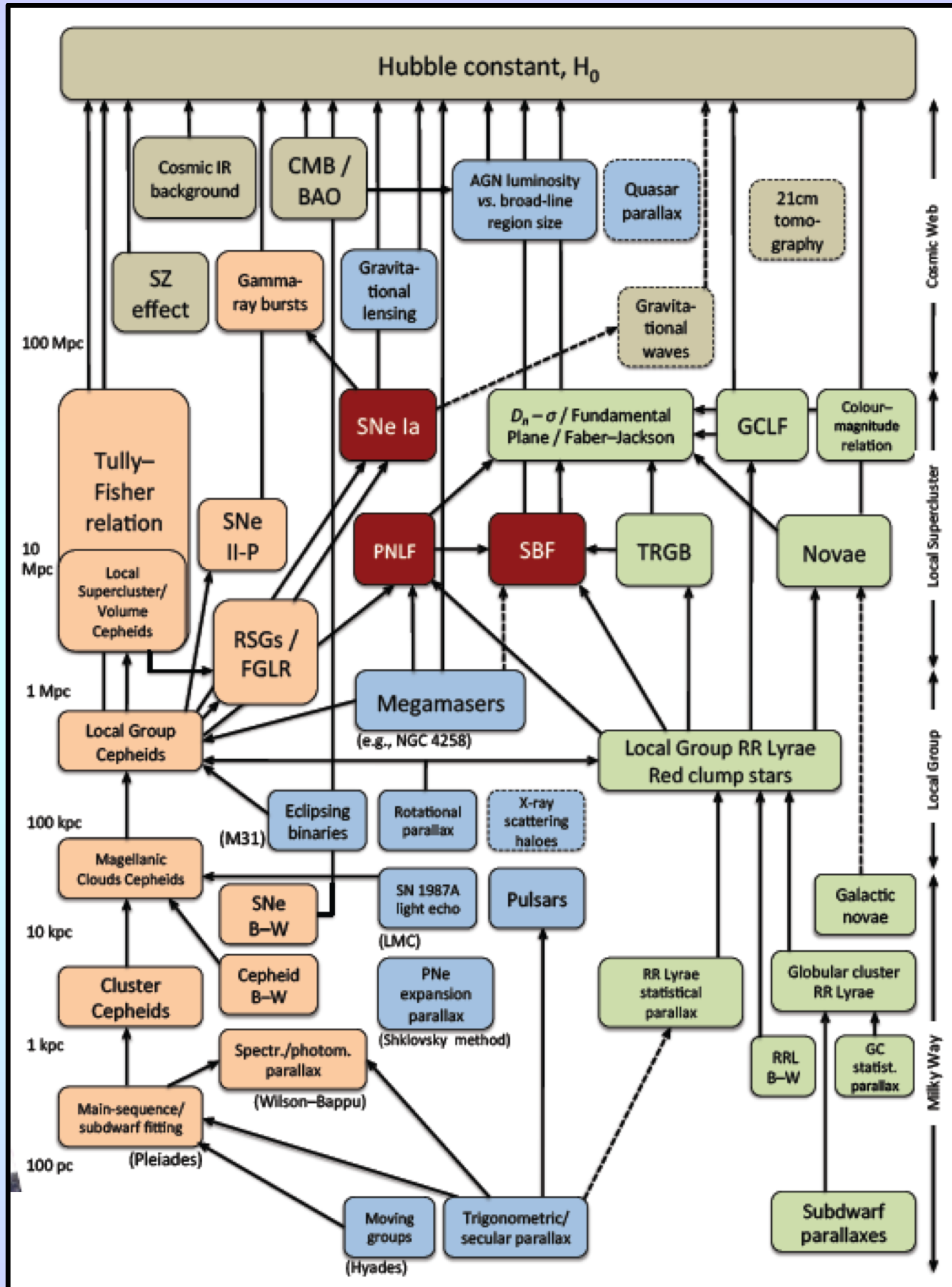
Hubble's original redshift-distance diagram

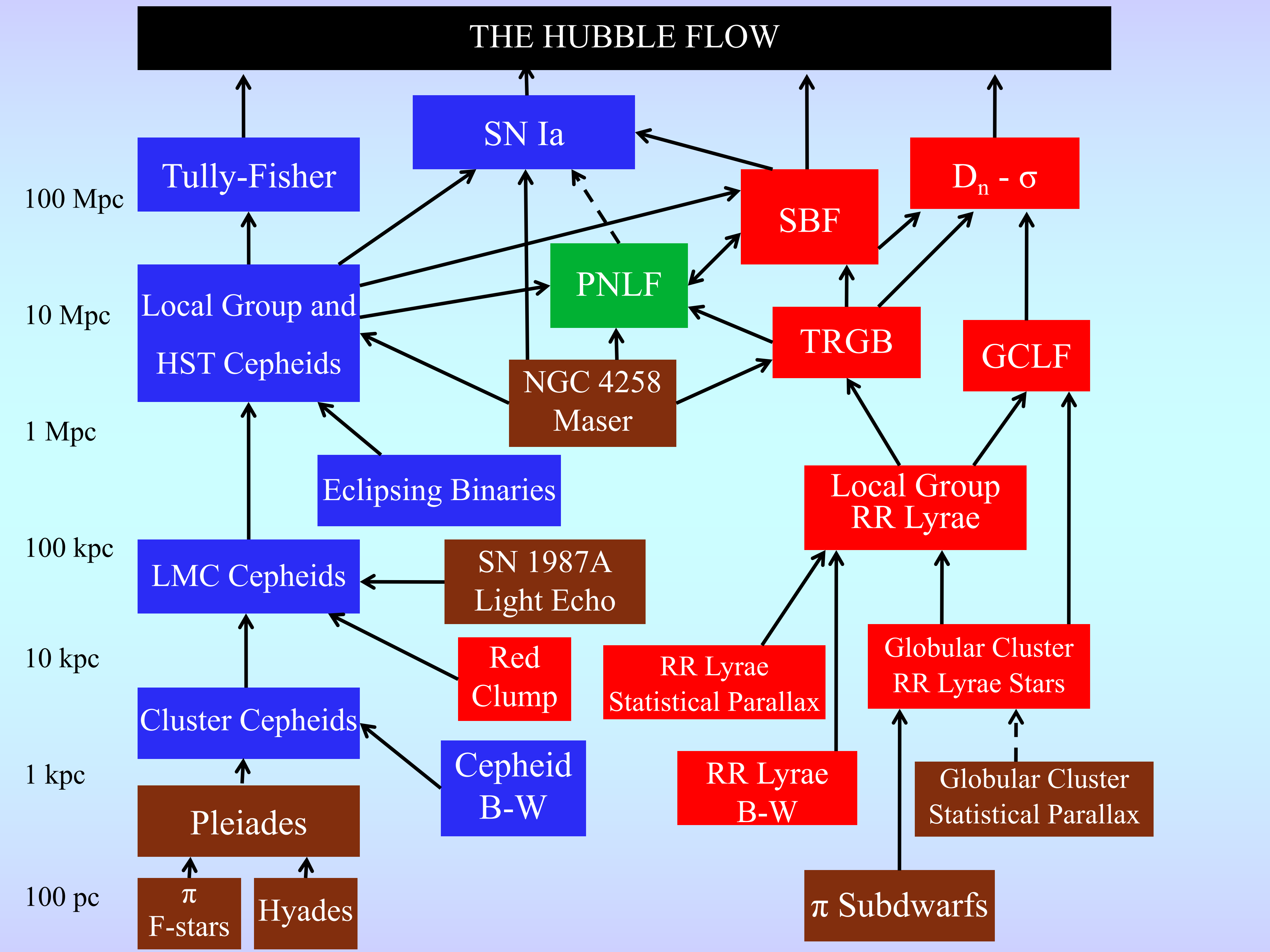
The Distance Ladder

In general, there are 3 ways to measure distance:

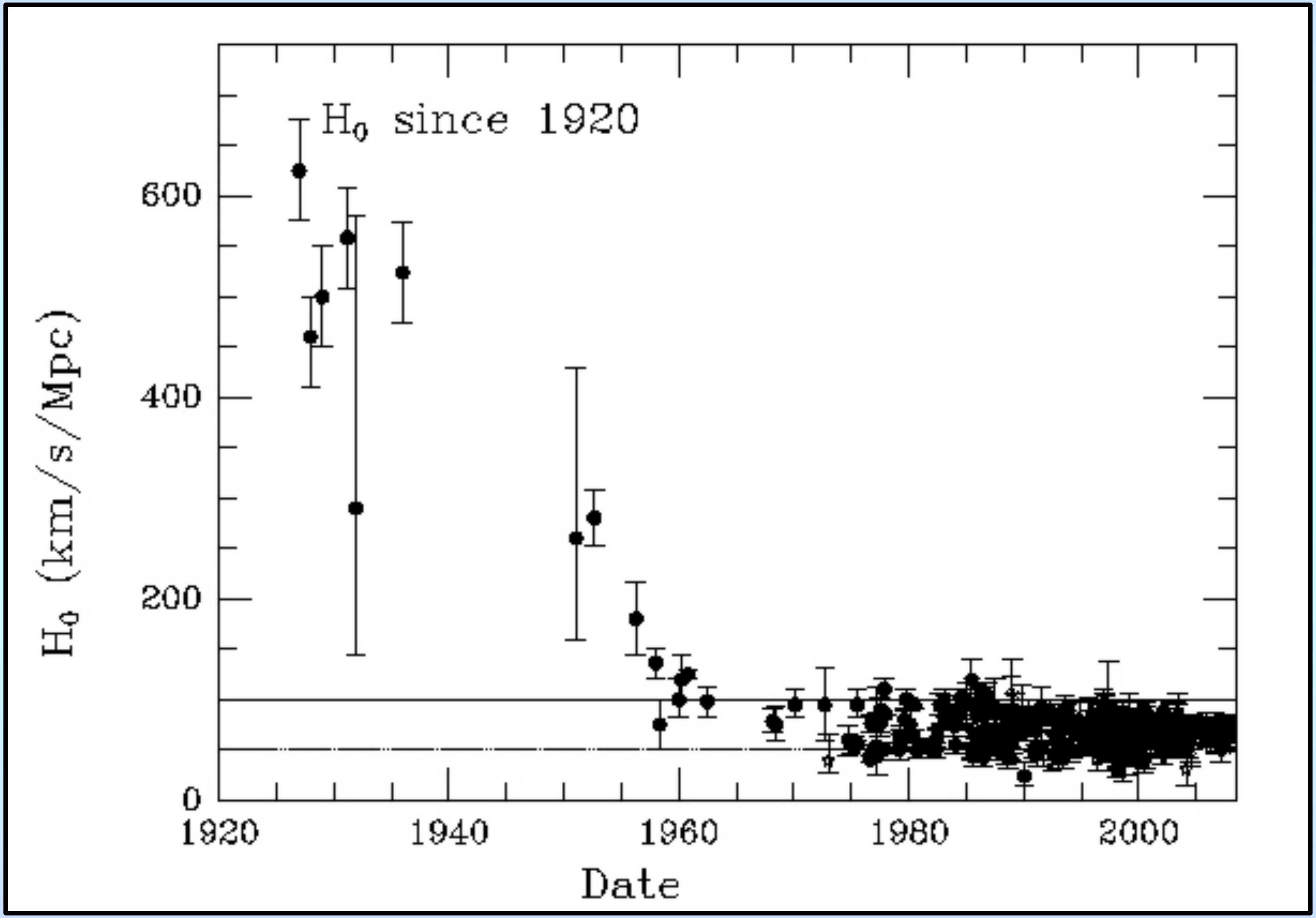
- Geometrically
- Via a standard ruler
- Via a standard candle

Standard rulers and candles require knowledge of the size/luminosity of the object (i.e., a calibration). Usually, these are derived from other techniques, thereby creating a distance ladder.




THE HUBBLE FLOW
100 Mpc
10 Mpc
1 Mpc
100 kpc
10 kpc
1 kpc
100 pc
Tully-Fisher
SN Ia
SBF
Dn - σ
Local Group and HST Cepheids
PNLF
TRGB
GCLF
NGC 4258 Maser
Eclipsing Binaries
Local Group RR Lyrae
LMC Cepheids
SN 1987A Light Echo
Red Clump
RR Lyrae Statistical Parallax
Globular Cluster RR Lyrae Stars
Cluster Cepheids
Cepheid B-W
RR Lyrae B-W
Globular Cluster Statistical Parallax
Pleiades
π F-stars
Hyades
π Subdwarfs

The Hubble Constant as a Function of Time



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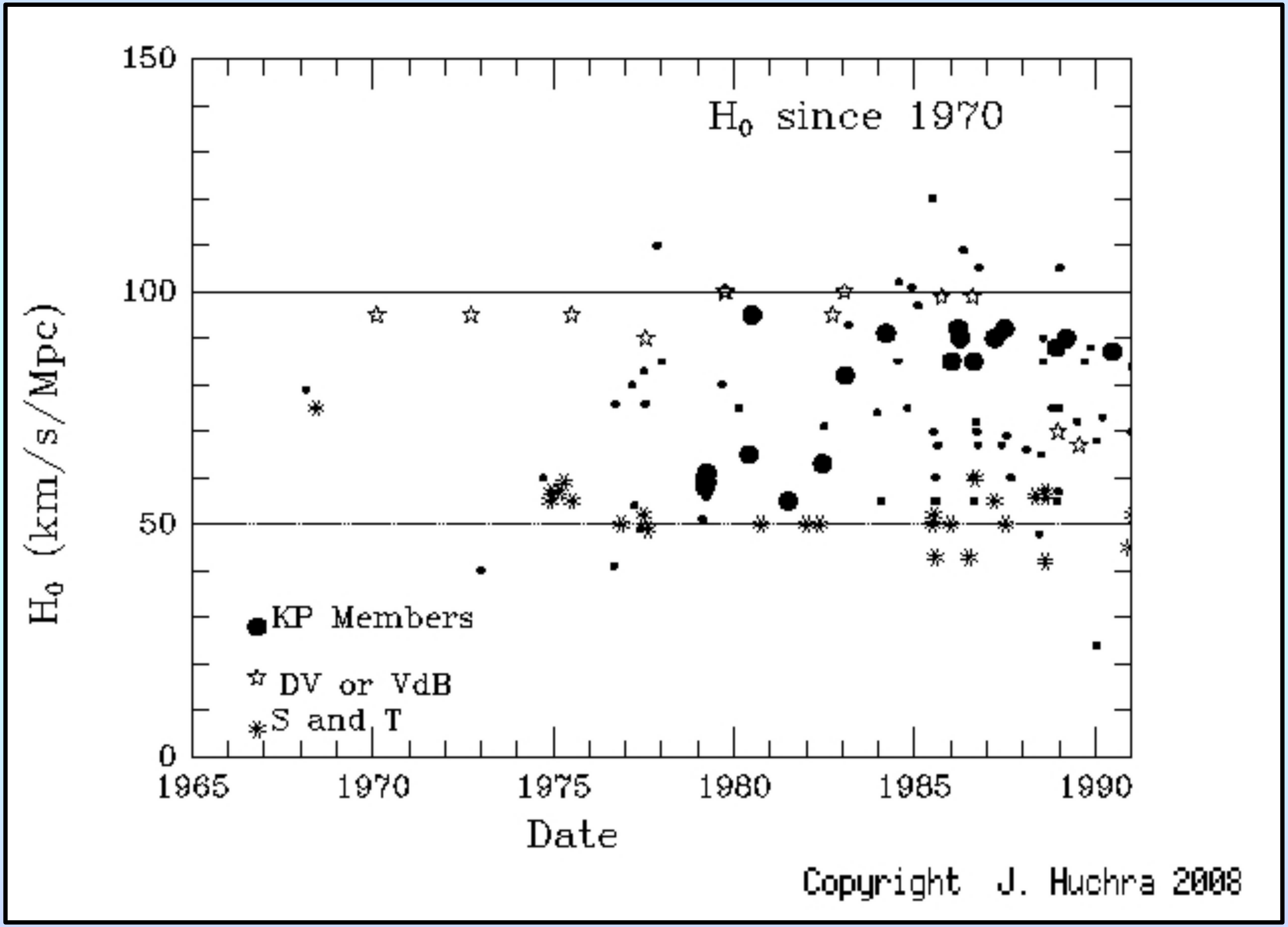


Geometric Methods

These methods depend on simple geometry and the movement of the Earth or an object in space.

Population I:

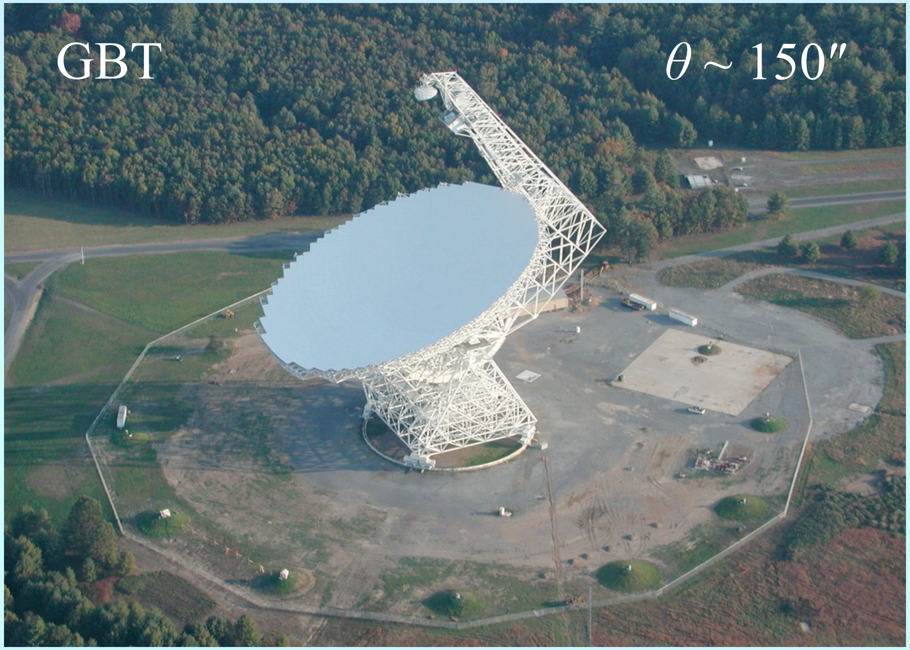
- Parallax of main-sequence stars
- Moving Cluster method (Hyades)
- SN 1987A Light Echo
- Mega-Masers

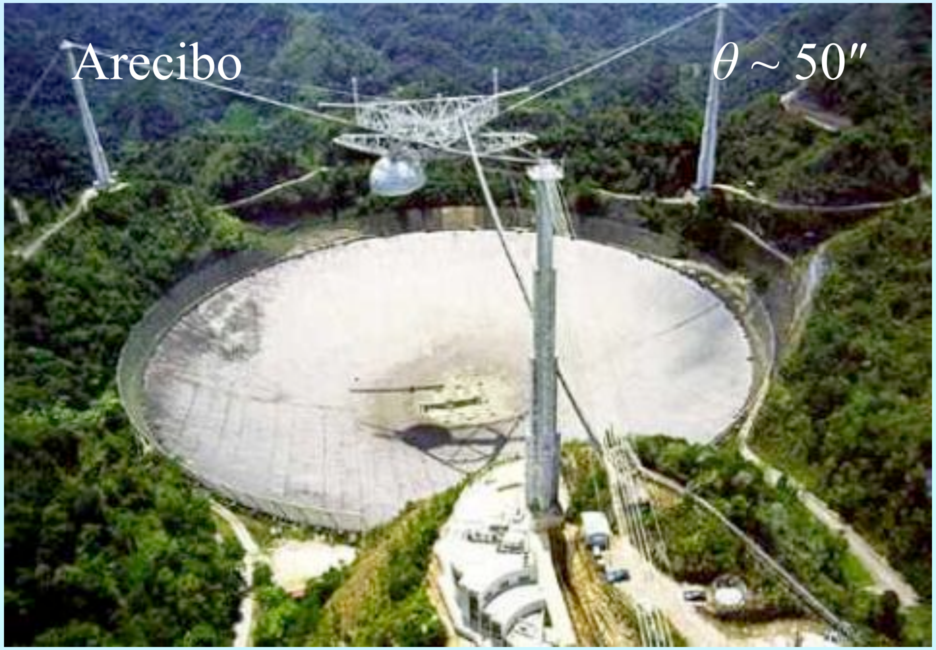
Population II:

- Globular Cluster Statistical Parallax
- Gaia Parallax to metal-poor stars
- (Statistical Parallax of metal-poor stars)

Aside: Diffraction Limit

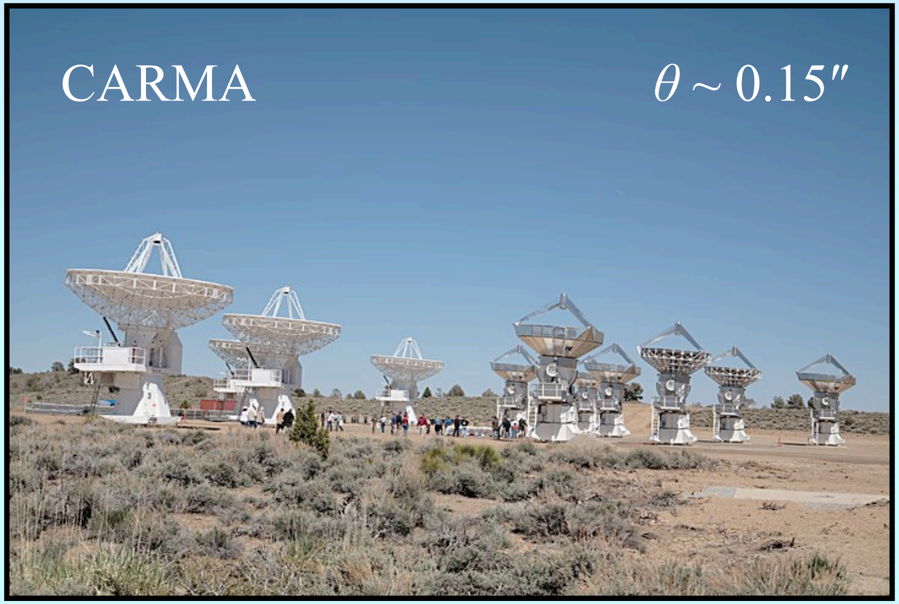
The diffraction limit of a telescope is $\theta \sim 1.22\ \lambda / D$. For single-dish radio telescopes, this is ~ 1′ or greater. For instance, at 5 GHz

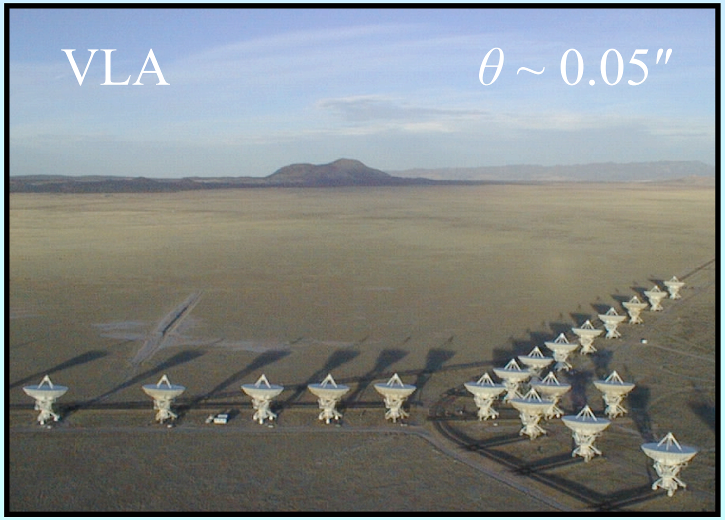


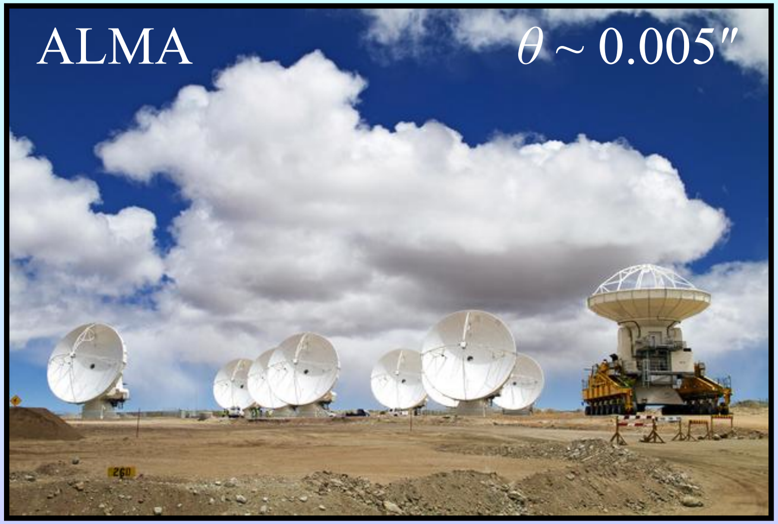


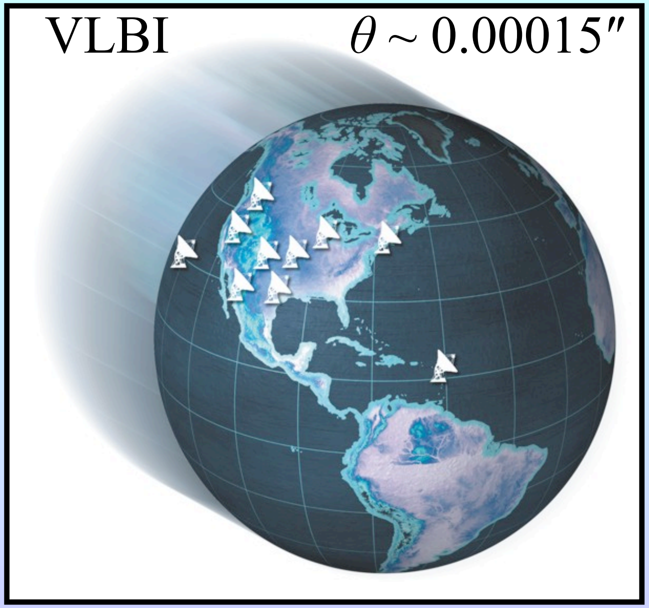
Aside: Diffraction Limit

However, to obtain a diffraction-limited image, a telescope need not be solid. The signal from several different apertures can be combined.



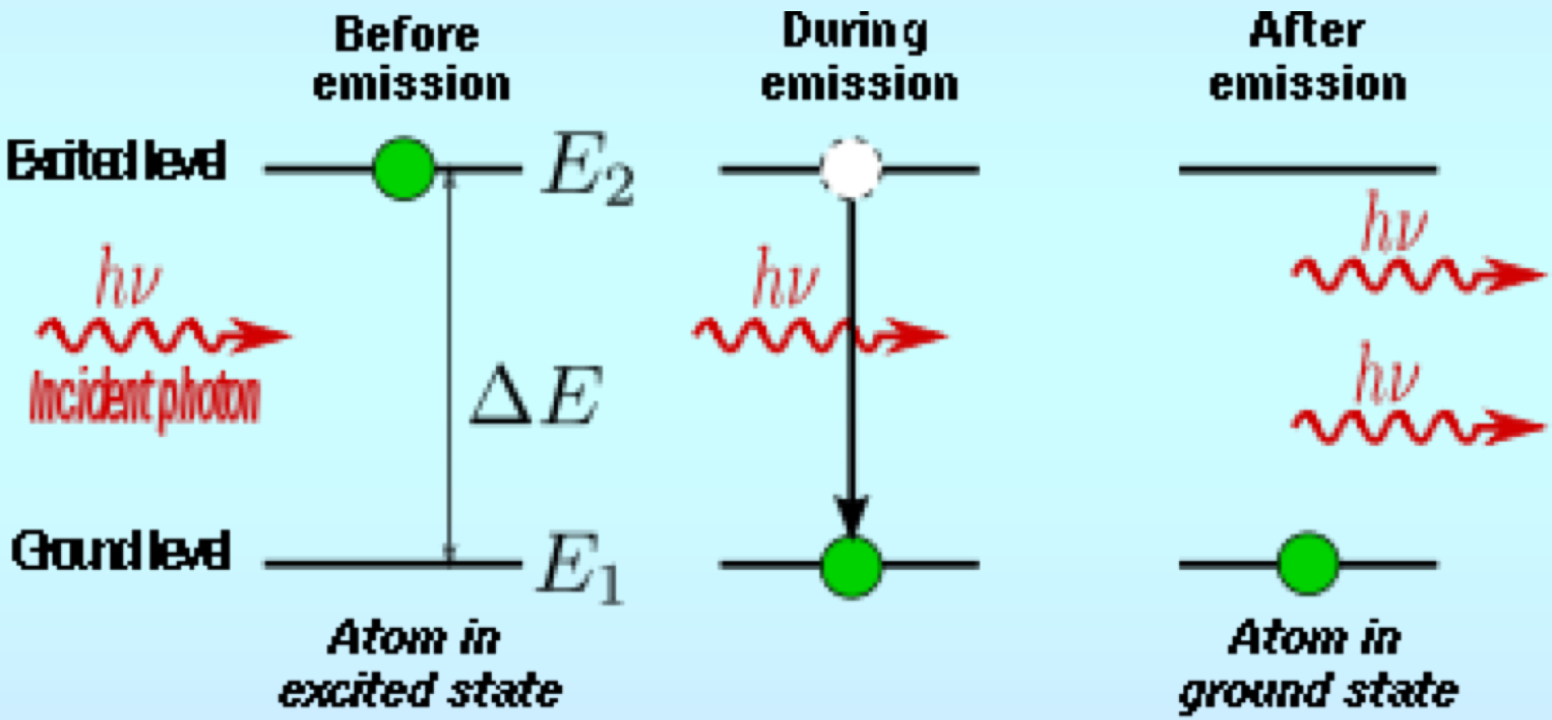






Mega-Masers

There is often gas orbiting about a galaxy's central black hole. In a few cases, the (long wavelength) emission from spontaneous decays can induce stimulated emission, amplifying the light. This is a maser.



$$E_2 - E_1 = \Delta E = h\nu$$

This light can be observed at very high spatial and spectral resolution.

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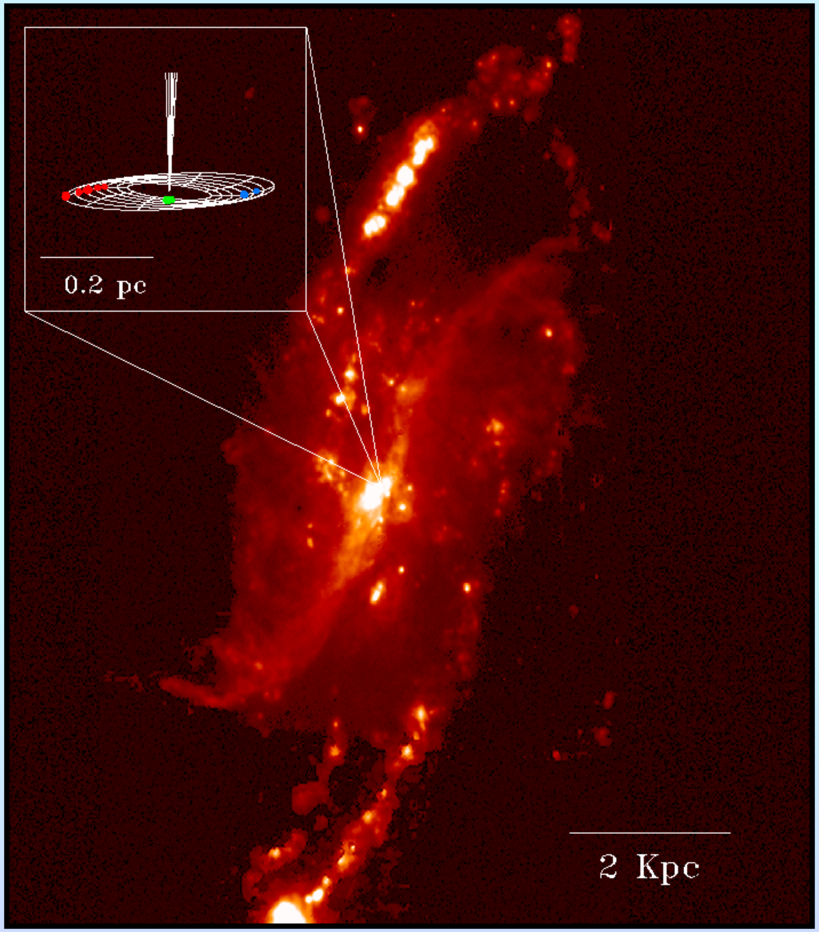
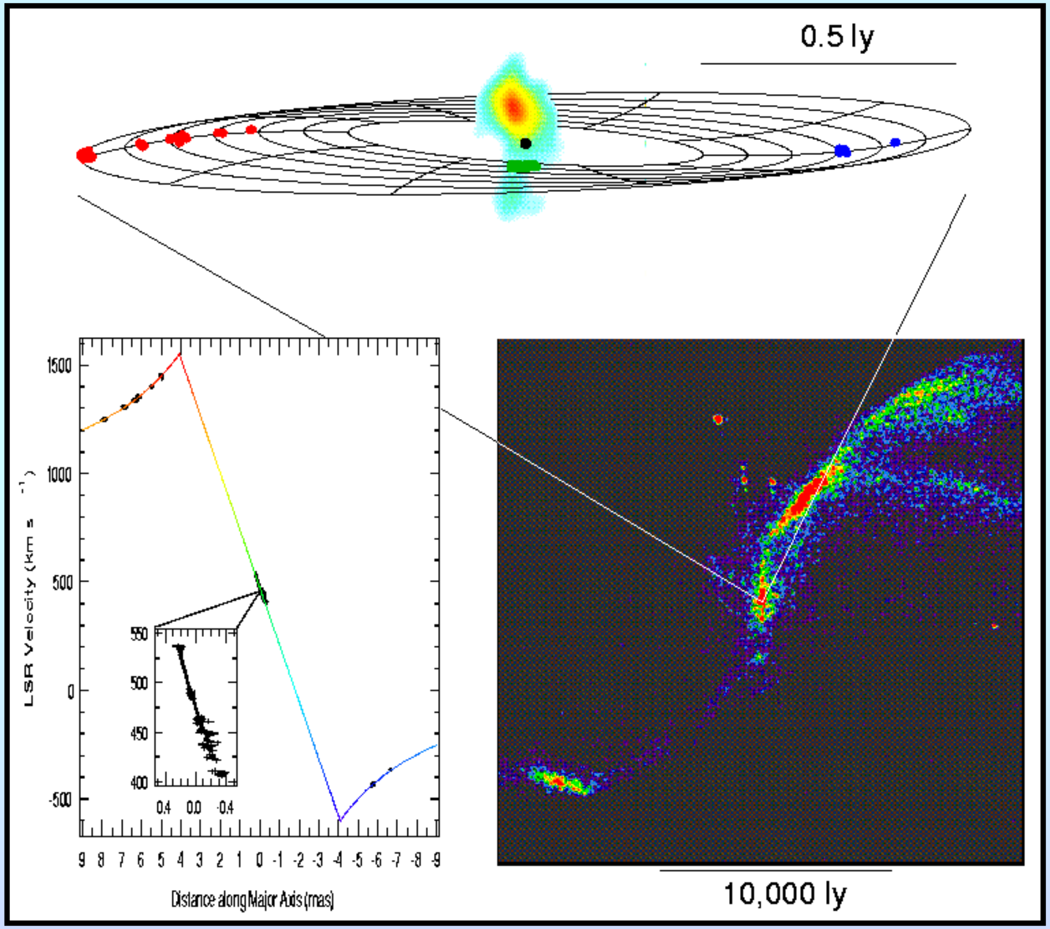




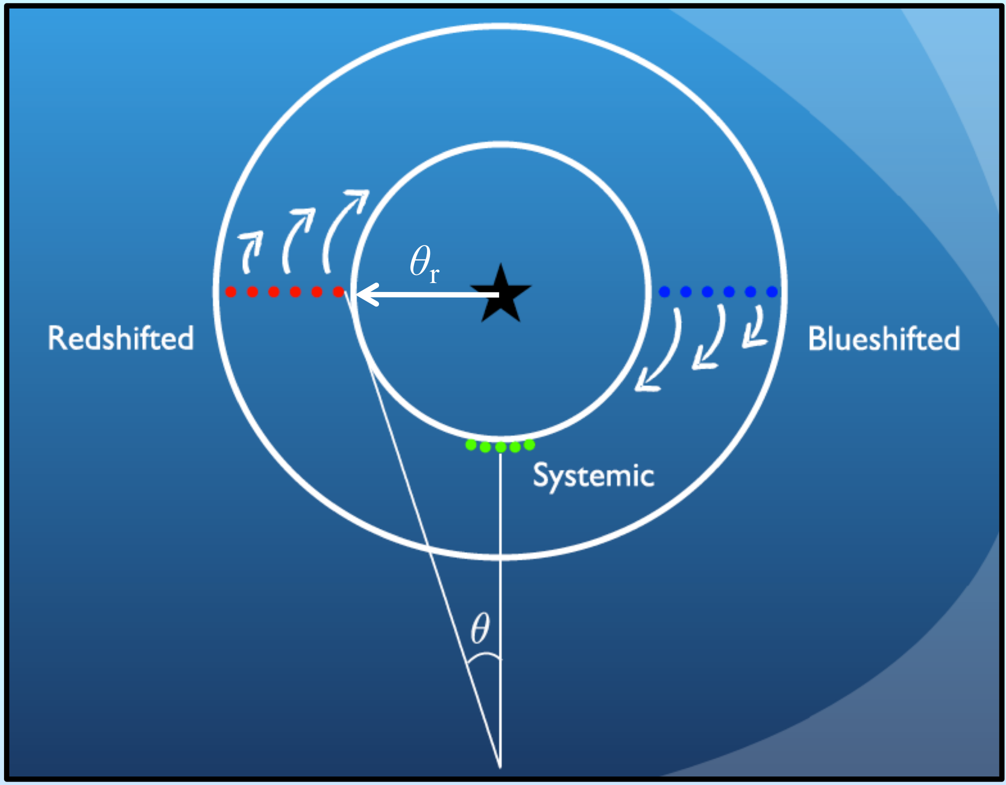
This light can be observed at very high spatial and spectral resolution.

Mega-Masers

Masers projected close to a black hole are "systemic" masers; masers projected farther out are "satellite" masers. For systemic masers one can observe proper motions:

$$\frac{GM}{r^2} = \frac{v^2}{r} \Rightarrow \frac{GM}{D\theta_r} = \left\{ D\frac{d\theta_r}{dt} \right\}^2 \Rightarrow$$

$$D \propto \left(\frac{M}{\theta_r} \right)^{1/3} \left(\frac{d\theta_r}{dt} \right)^{-2/3}$$



Meanwhile, the rotation curve of the satellite masers is

$$v^2 = \frac{GM}{r} = \frac{GM}{D\theta_s} \Rightarrow D \propto \frac{M}{v^2 \theta_s}$$

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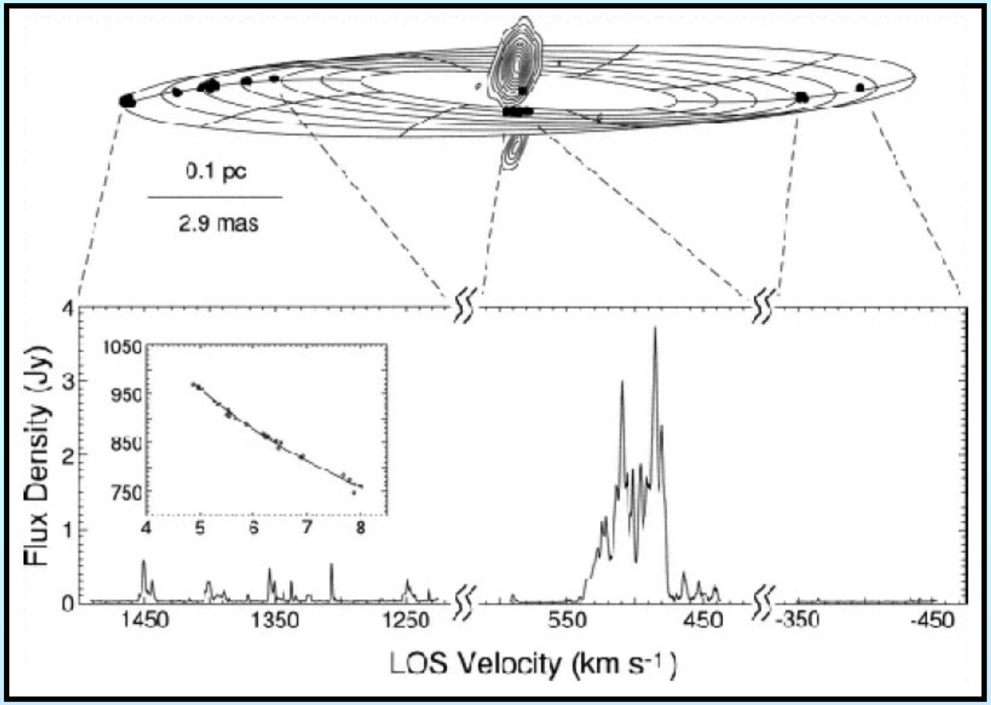

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By monitoring the systemic masers and tracing the velocity curve of the satellites, the distance (and black hole mass) can be measured to high precision.

Standard Candles

Standard candles rely on the inverse square law of light, and therefore can be affected by dust. All distances based on these methods must (ideally) also quote the assumed extinction.

Population I:

- Cepheids
- Tully-Fisher Relation
- SN Ia

Population II:

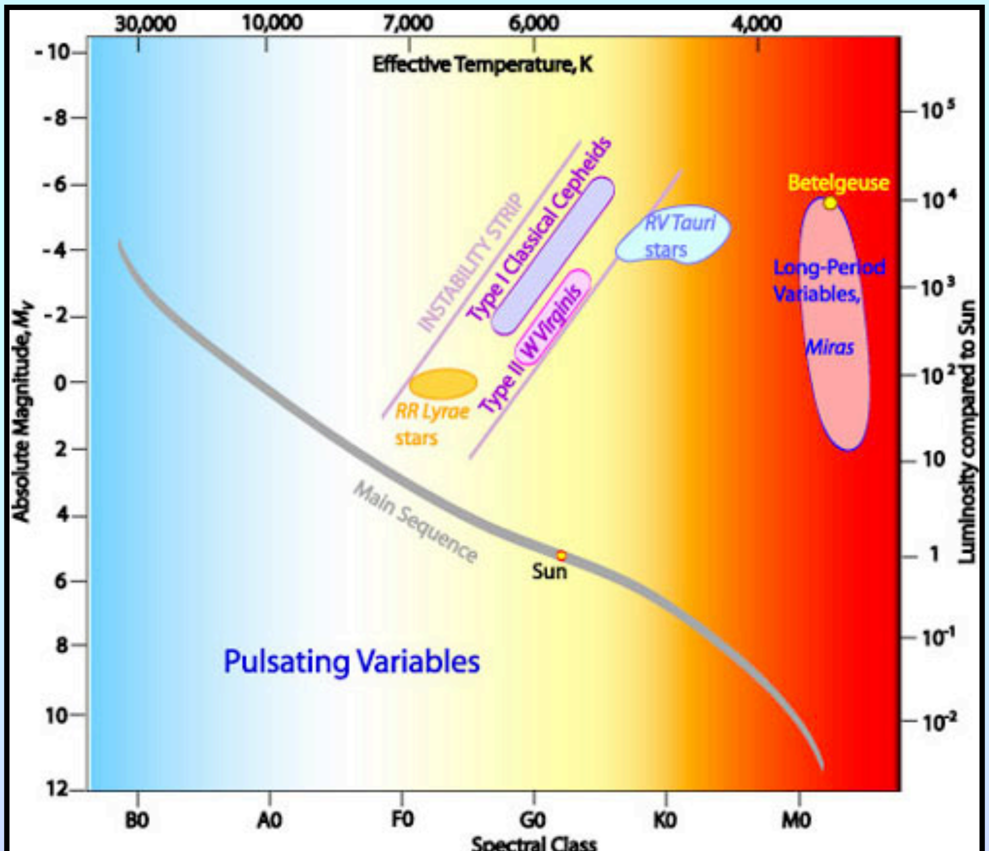
- RR Lyrae Stars
- Tip of the Red Giant Branch
- Surface Brightness Fluctuations
- Globular Cluster Luminosity Function

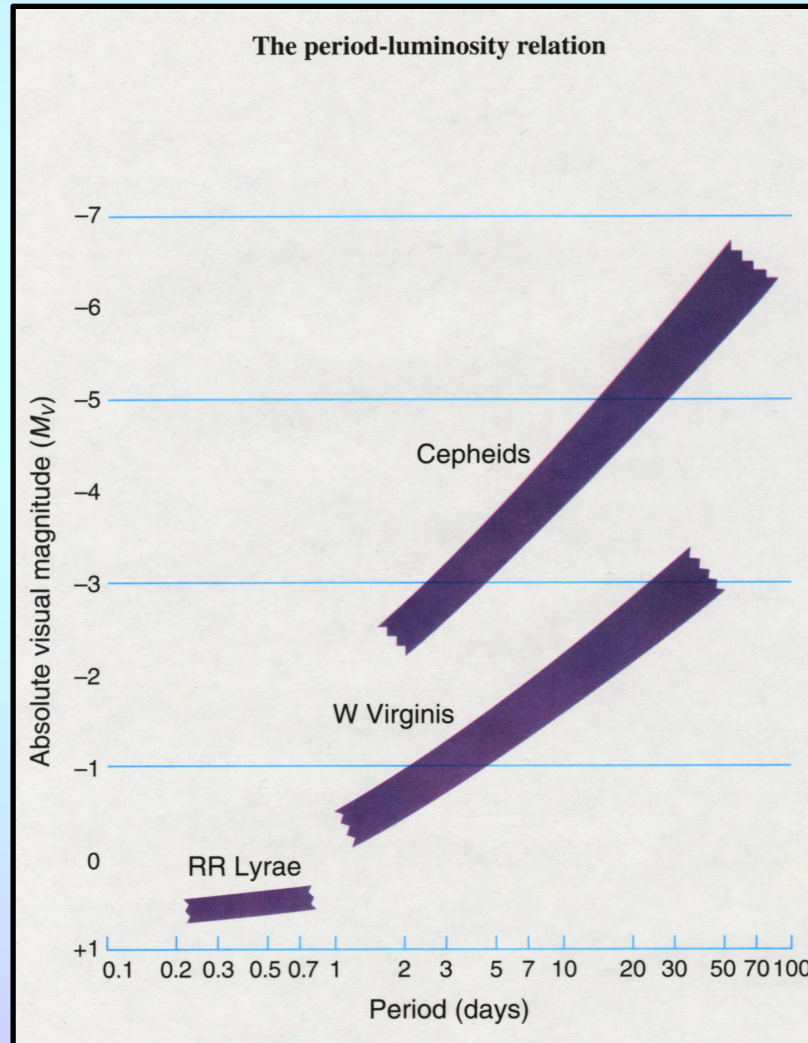
Both

- Planetary Nebula Luminosity Function

Pulsating Stars

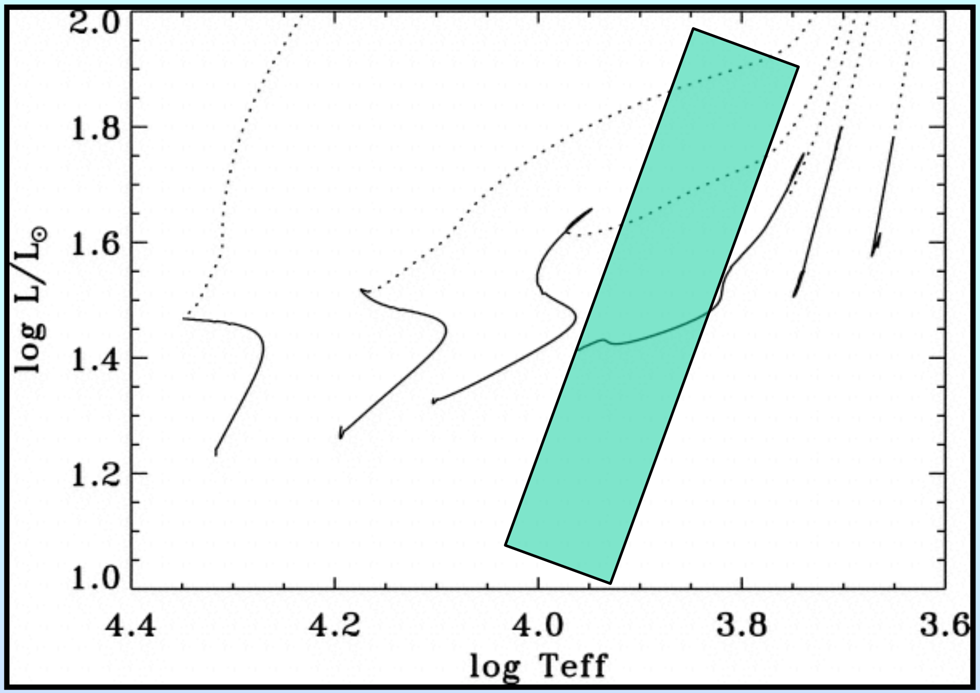
Because the physics of the instability strip is reasonably well understood, most types of pulsating stars can be used to derive distance. Measurements of RR Lyrae stars and Cepheids are by far the most common.

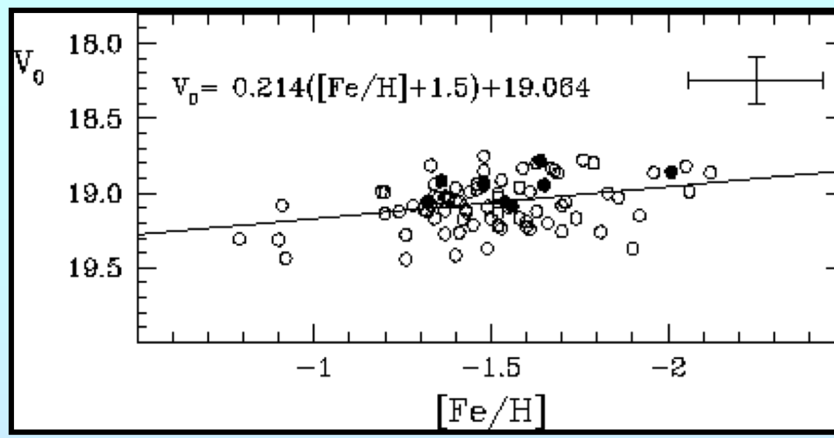




RR Lyrae Stars

Within a globular cluster, all the RR Lyrae stars have the same (mean) luminosity (at least in optical light). However, as stars evolve off the zero-age horizontal branch, they may also go through the instability strip. These *W Virginis* variables are brighter and have longer periods (but can be confused with RR Lyrae). $\langle M_V \rangle$ for RR Lyrae probably depends on metallicity as well.

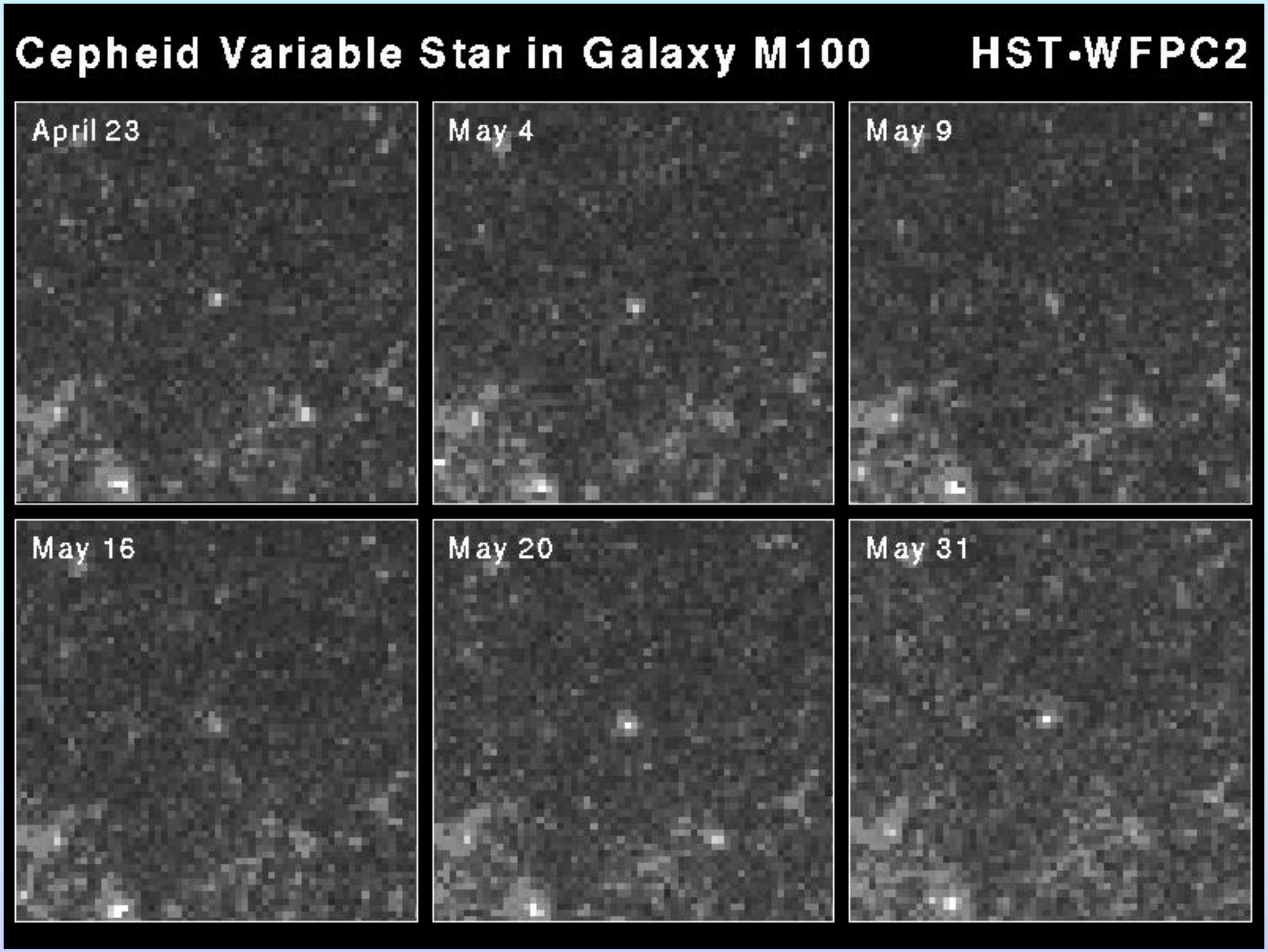




The RR Lyrae star distance to our nearest galaxy (the Large Magellanic Cloud) differs by ~5% from that derived from Cepheids.

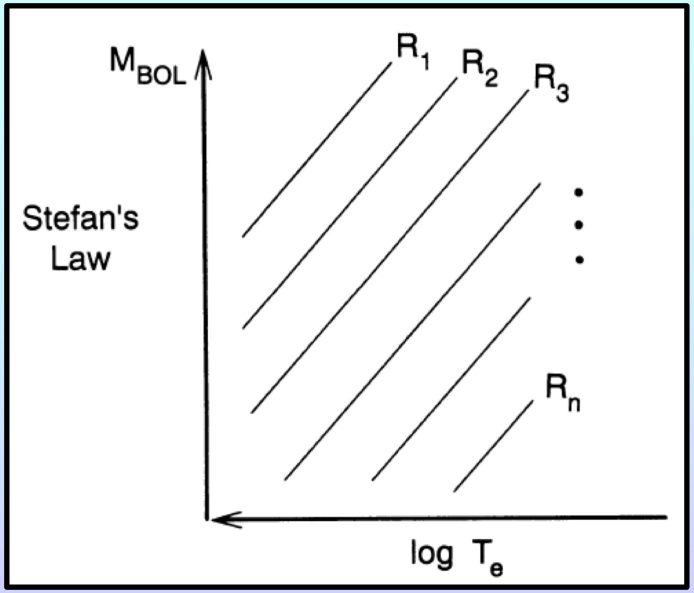
Cepheids

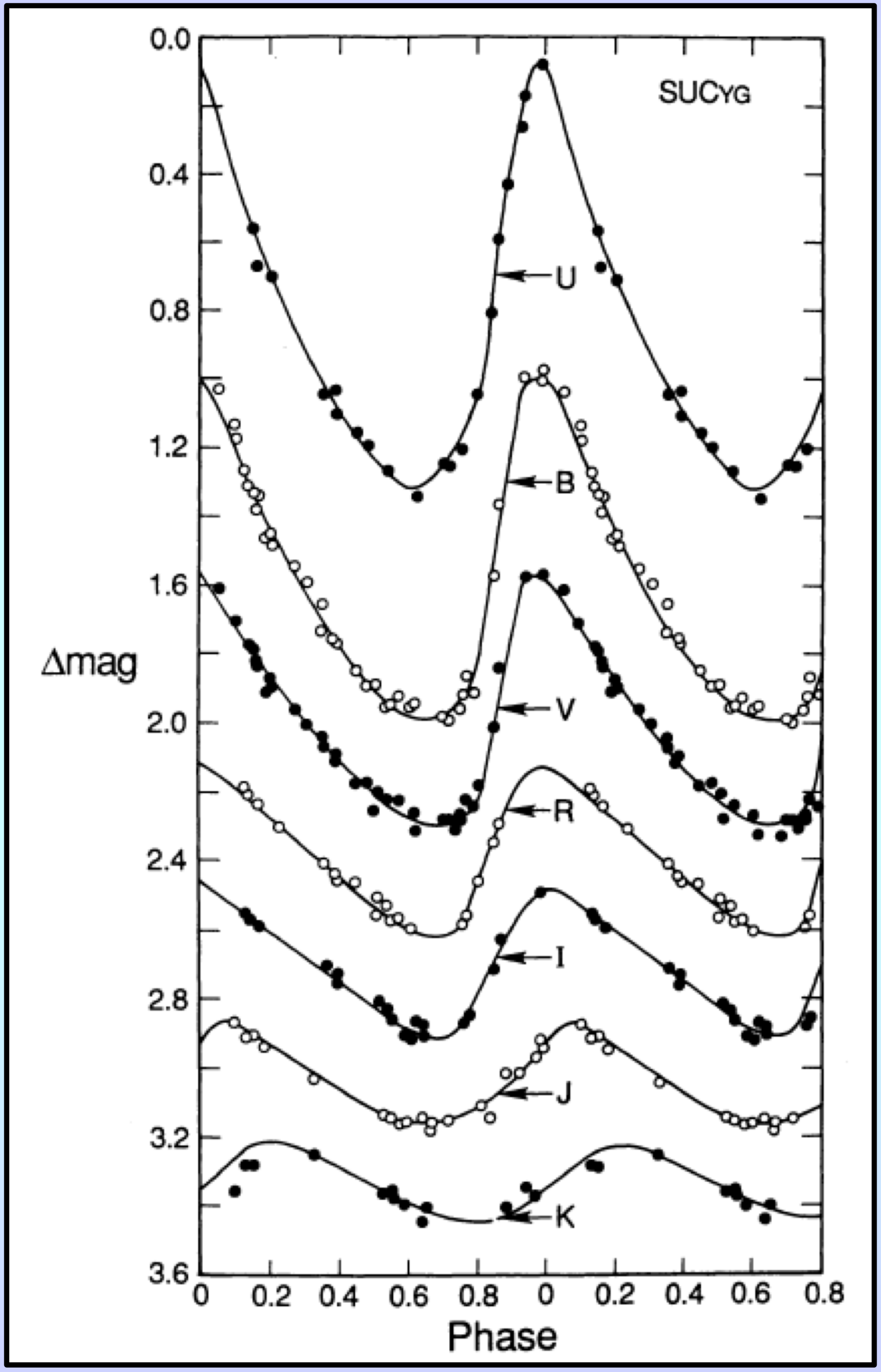
Luminous, high mass "blue loop" stars in the instability strip in the H-R diagram are Cepheids, and can be see by the Hubble Space Telescope out to ~ 25 Mpc. Cepheids are the easiest to use.



Cepheid Pulsations

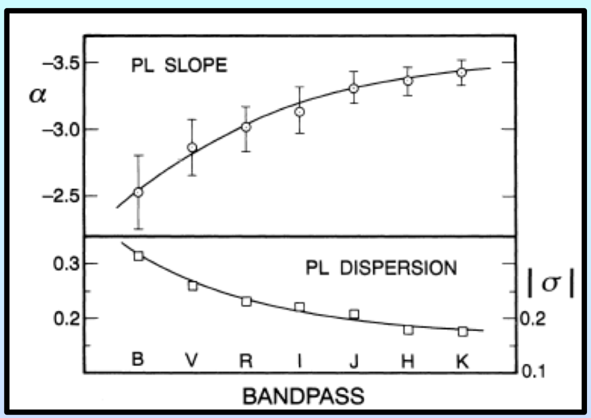
As a Cepheid pulsates, it changes both its radius and temperature, i.e., $L = 4 \pi R^2 \sigma T^4$. In the IR the temperature change isn't important, since that's the Raleigh-Jeans tail of the blackbody distribution. Optical wavelengths see changes in both R and T, and thus the variations have greater amplitude.

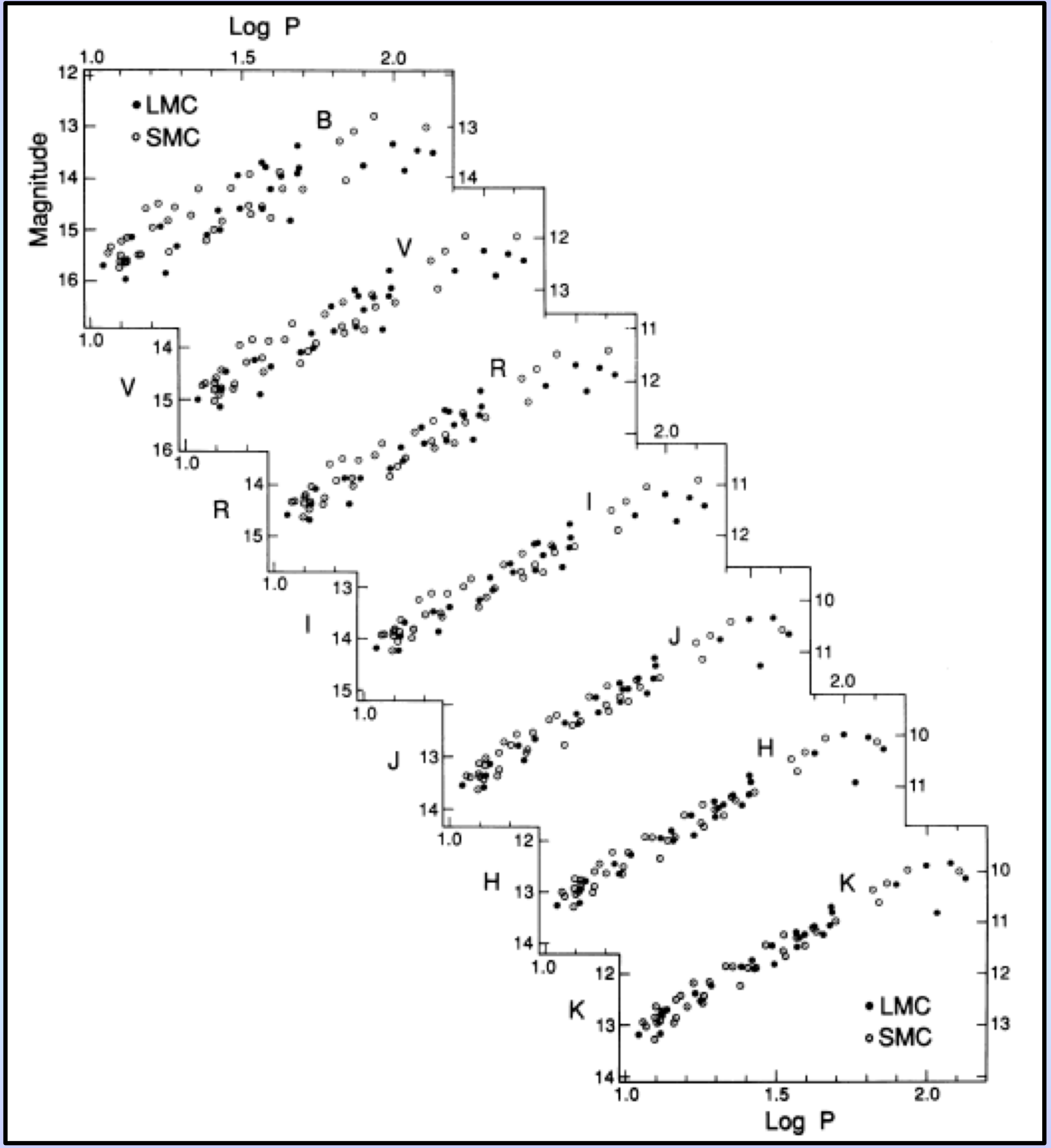




Cepheids

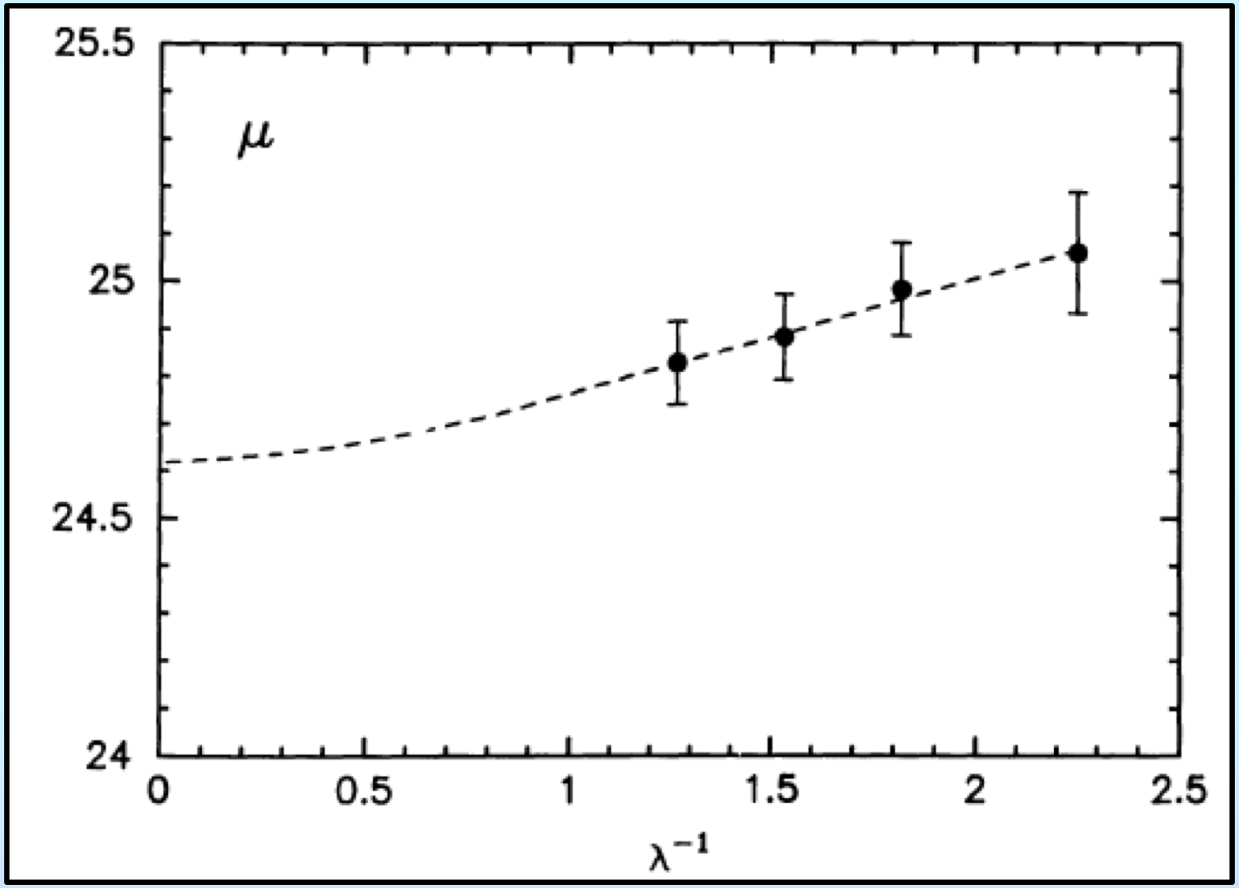
Cepheids are Pop I objects. At least partially due to dust, the scatter in the period - luminosity distribution is greater at shorter wave-lengths.





Cepheids

Cepheids magnitudes are affected by extinction, but the IR is affected less than the optical. By measuring Cepheids in multiple bandpasses, you can solve for distance and extinction.



$$\mu_0 = \mu_\lambda - R_\lambda\, E(B\text{-}V)$$

Aside: the Wesenheit

Although it is not used that often, there is a way to make your data reddening-independent. Instead of measuring a star's brightness in B or R or some other filter, measure it in

$$W = V - R_V(B - V)$$

where V and *(B-V)* are the object's observed magnitude and color, and R_V is the ratio of total to differential reddening, i.e., $A_V = R_V E(B\text{-}V)$. The differential reddening is defined as $E(B\text{-}V) = (B\text{-}V) - (B\text{-}V)_0$, where the subscript zero represents the *true* (non-extincted) measurement. So

$$\begin{aligned} W &= V - R_V(B - V) \\ &= V_0 + A_V - R_V\left\{(B - V)_0 + E(B - V)\right\} \\ &= V_0 + R_V E(B - V) - R_V(B - V)_0 - R_V E(B - V) \\ &= V_0 - R_V(B - V)_0 \qquad \text{so} \end{aligned}$$

$$W = V - R_V(B - V) = V_0 - R_V(B - V)_0$$

This "Wesenheit" is therefore independent of reddening!

Tully-Fisher Relation

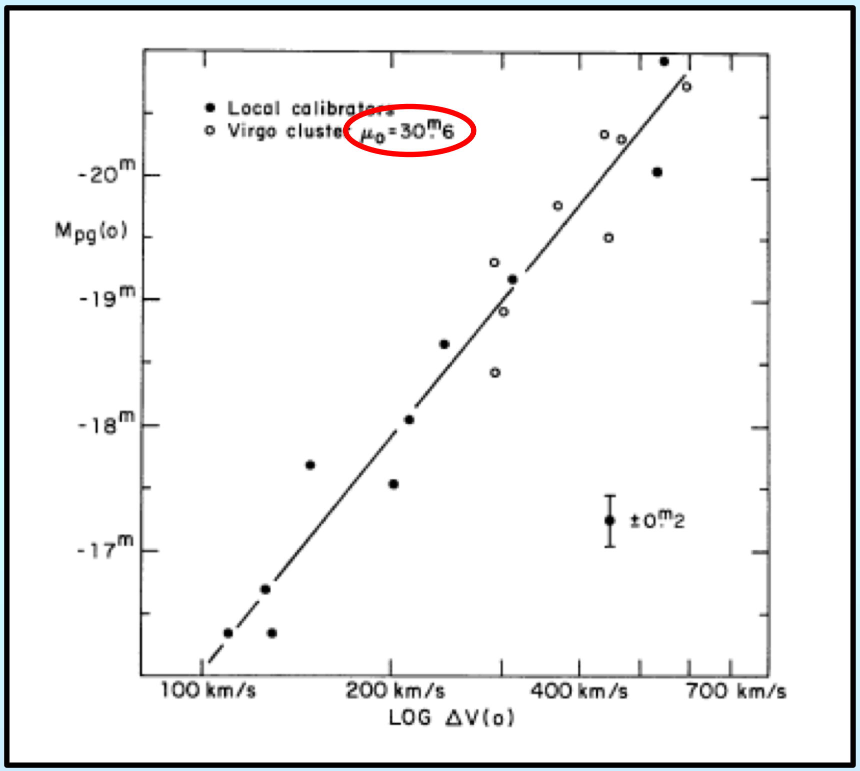
The rotation speed of a spiral galaxy (which is proportional to the line-width of its 21 cm H I emission line) is related to its luminosity by $M = \mathrm{a} + \log W$. Since line-width is distance independent, this immediate turns spiral galaxies into a standard candle.

Note: W must reflect the true rotation velocity. One needs to correct for inclination, i.e., $v_{\mathrm{true}} = v_{\mathrm{obs}} / \sin i$.

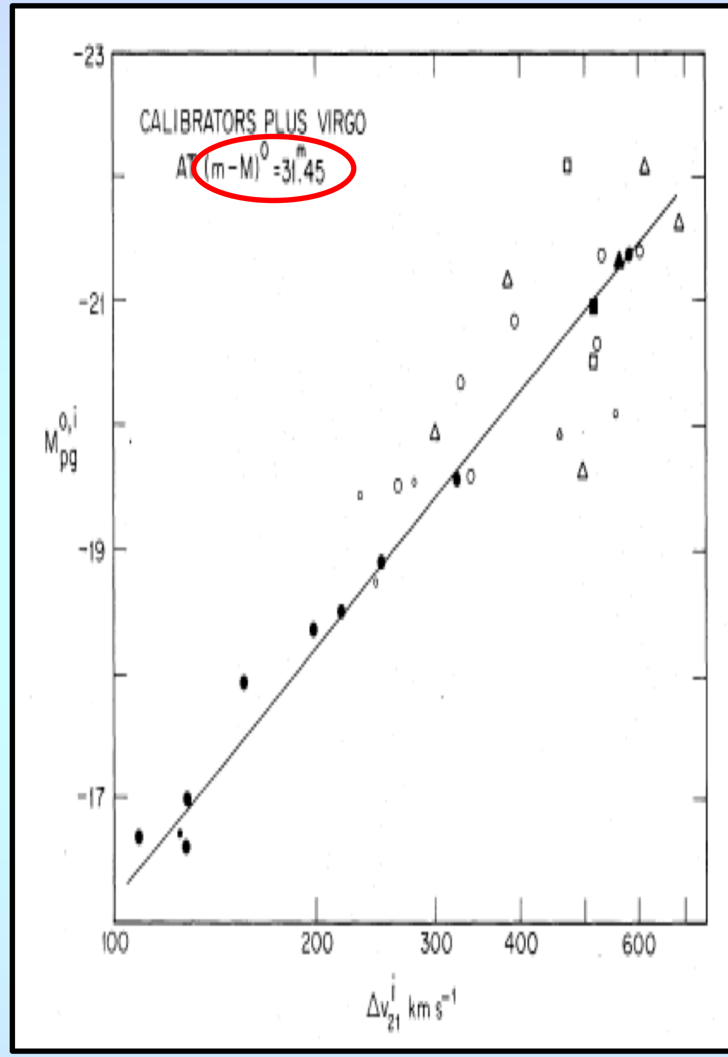
Historical Note: In the past, Tully-Fisher measurements have been extremely controversial. Different authors have used the <u>same technique with identical data</u> to derive distances that differ by a factor of 2!

Tully-Fisher Relation

Note the difference in distance modulus to the Virgo Cluster.



Original: Tully & Fisher (1977)



Rebuttal: Sandage & Tammann (1976)

Why the difference? The devil is in the details!!!

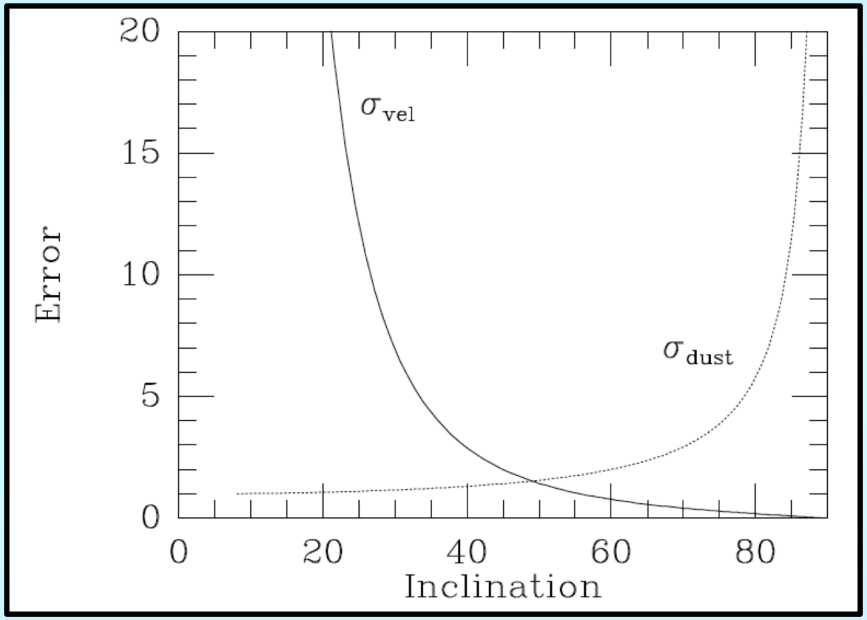
Tully-Fisher and Inclination

First, consider propagation of errors. The inclination of a flat disk is related to the axis ratio by cos $i = b/a = \varepsilon$. The true rotation speed is therefore related to the observed speed by

$$v_{\text{true}} = \frac{v_{\text{obs}}}{\sin i} = \frac{v_{\text{obs}}}{\left(1-\cos^2 i\right)^{1/2}} = \frac{v_{\text{obs}}}{\left(1-\varepsilon^2\right)^{1/2}}$$

Now according to error propagation

$$\sigma_{v_{true}}^2 = \sigma_{v_{\text{obs}}}^2 \left(\frac{\partial v_{\text{true}}}{\partial \varepsilon}\right)^2 = \sigma_\varepsilon^2 v_{\text{obs}}^2 \left\{\frac{\varepsilon}{\left(1-\varepsilon^2\right)^{3/2}}\right\}^2$$



The errors get very large as the galaxy becomes more face-on. Tully-Fisher distances for systems more face-on than ~ 45° are totally unreliable.

Malmquist Bias

Astronomical observations are usually biased. The most famous bias comes from Malmquist (1922), who was attempting to measure the brightness of F-stars in the Galaxy. Suppose we were to observe (perfectly) all F-stars brighter than a certain apparent magnitude. Now suppose that intrinsically, there is some dispersion in F-star magnitudes, such that there is a mean magnitude M_0 and a Gaussian dispersion, σ. In other words, the true luminosity function of F-stars is

$$\Phi(M) = \Phi_0 \exp\left\{ -\frac{(M - M_0)^2}{2\sigma^2} \right\}$$

The observed distribution of absolute magnitudes will then be

$$A(M) = \int_0^\infty \Phi(M)\rho(r) \cdot 4\pi r^2 dr \propto \int_0^\infty \Phi_0 \exp\left\{ -\frac{(M - M_0)^2}{2\sigma^2} \right\} \rho(r) r^2 dr$$

Now let's take the derivative of this

Malmquist Bias

$$\frac{dA(M)}{dM} = \int_0^\infty \Phi_0 \frac{d}{dM}\left[\exp\left\{-\frac{\left(M-M_0\right)^2}{2\sigma^2}\right\}\right]\rho(r)r^2dr$$

$$= \int_0^\infty \Phi_0 \left\{-\frac{2\left(M-M_0\right)}{2\sigma^2}\right\} \exp\left\{-\frac{\left(M-M_0\right)^2}{2\sigma^2}\right\}\rho(r)r^2dr$$

$$= \int_0^\infty \frac{M_0}{\sigma^2}\Phi(M)\rho(r)r^2dr - \int_0^\infty \frac{M}{\sigma^2}\Phi(M)\rho(r)r^2dr$$

$$= \frac{M_0}{\sigma^2}A(M) - \frac{1}{\sigma^2}\int_0^\infty M\,\Phi(M)\rho(r)r^2dr$$

$$\frac{dA(M)}{dM}\frac{1}{A(M)} = \frac{1}{\sigma^2}\left\{M_0 - \frac{\int M\Phi(M)\rho(r)r^2\,dr}{\int \Phi(M)\rho(r)r^2\,dr}\right\} = \frac{1}{\sigma^2}\left(M_0 - \langle M\rangle\right)$$

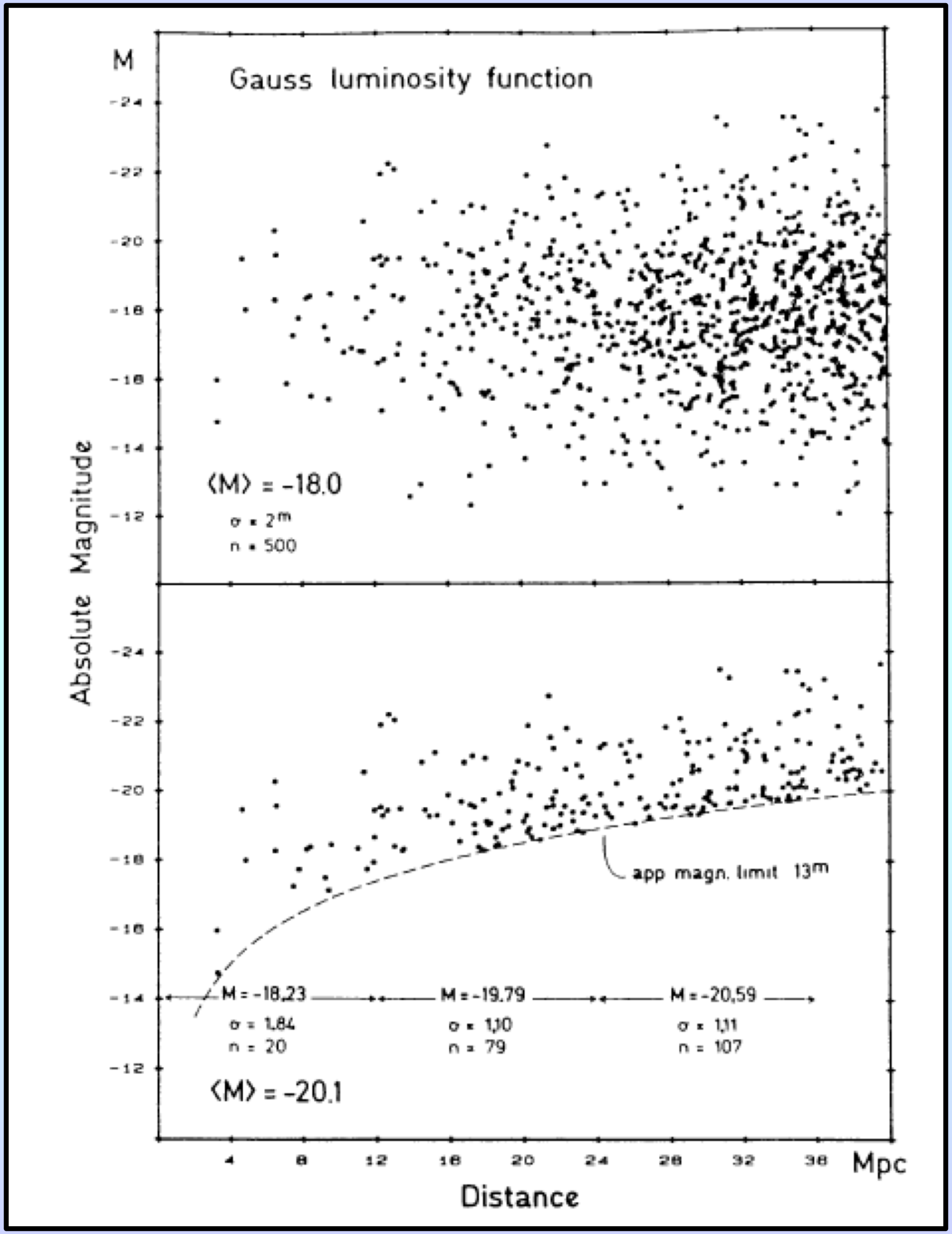
Since the left side of the equal sign is not zero, the mean magnitude you observe, $\langle M\rangle$, can not be the mean magnitude of the distribution! The difference is proportional to σ^2.

Malmquist Bias

$$M_0 - \langle M \rangle = \sigma^2 \cdot \frac{d \ln A(m)}{dm}$$

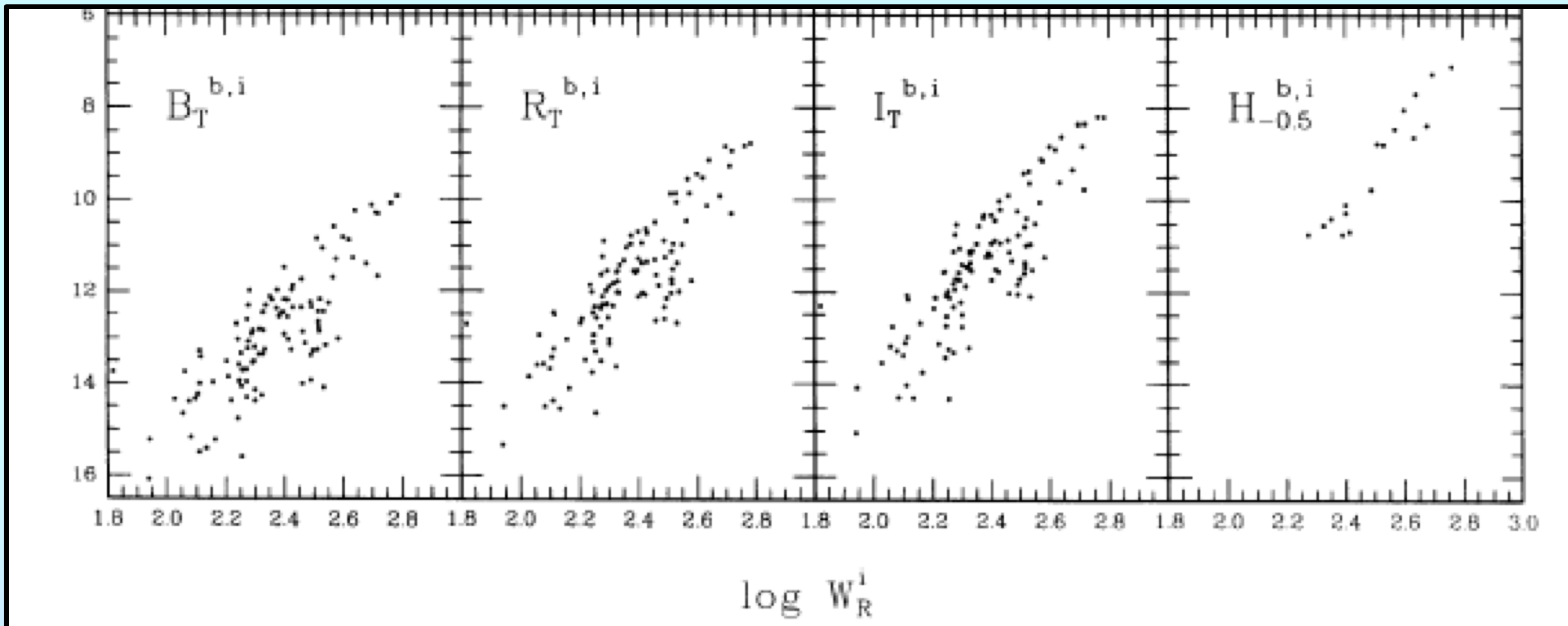
The offset occurs because of the sample's magnitude cutoff. Objects with brighter than average magnitudes will be included in the sample, while fainter objects are not. To correct for this, you need to know the intrinsic dispersion of the sample. But the same effect that makes $M_0 \neq \langle M \rangle$ causes $\sigma \neq \sigma_{meas}$. In particular

$$\sigma_{meas}^2 = \sigma^2 \cdot \left\{ 1 + \sigma^2 \frac{d^2 \ln A}{dm^2} \right\}$$



Tully-Fisher Distances

For example, the largest nearby cluster of galaxies is in Virgo. Is the scatter seen in the Tully-Fisher distances due to an intrinsic dispersion of σ ~ 0.7 mag, or does the cluster have depth along the line-of-sight, so that σ is smaller? What you think makes a factor of ~ 2 difference!

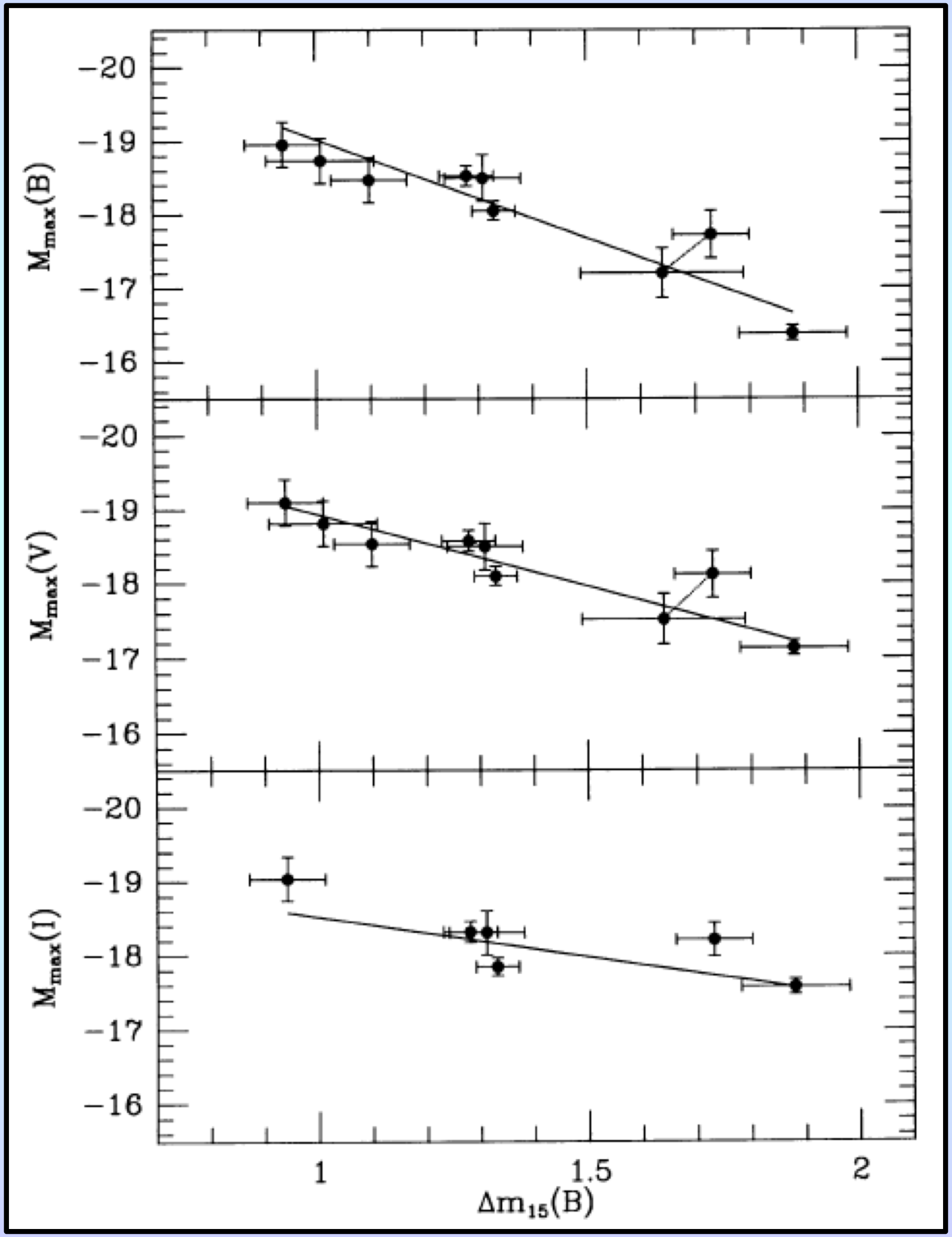


Cepheid observations eventually proved that σ ~ 0.3 mag was closer to the truth, and Virgo is elongated along the line-of-sight.

SN Ia Maximum Magnitude – Rate of Decline

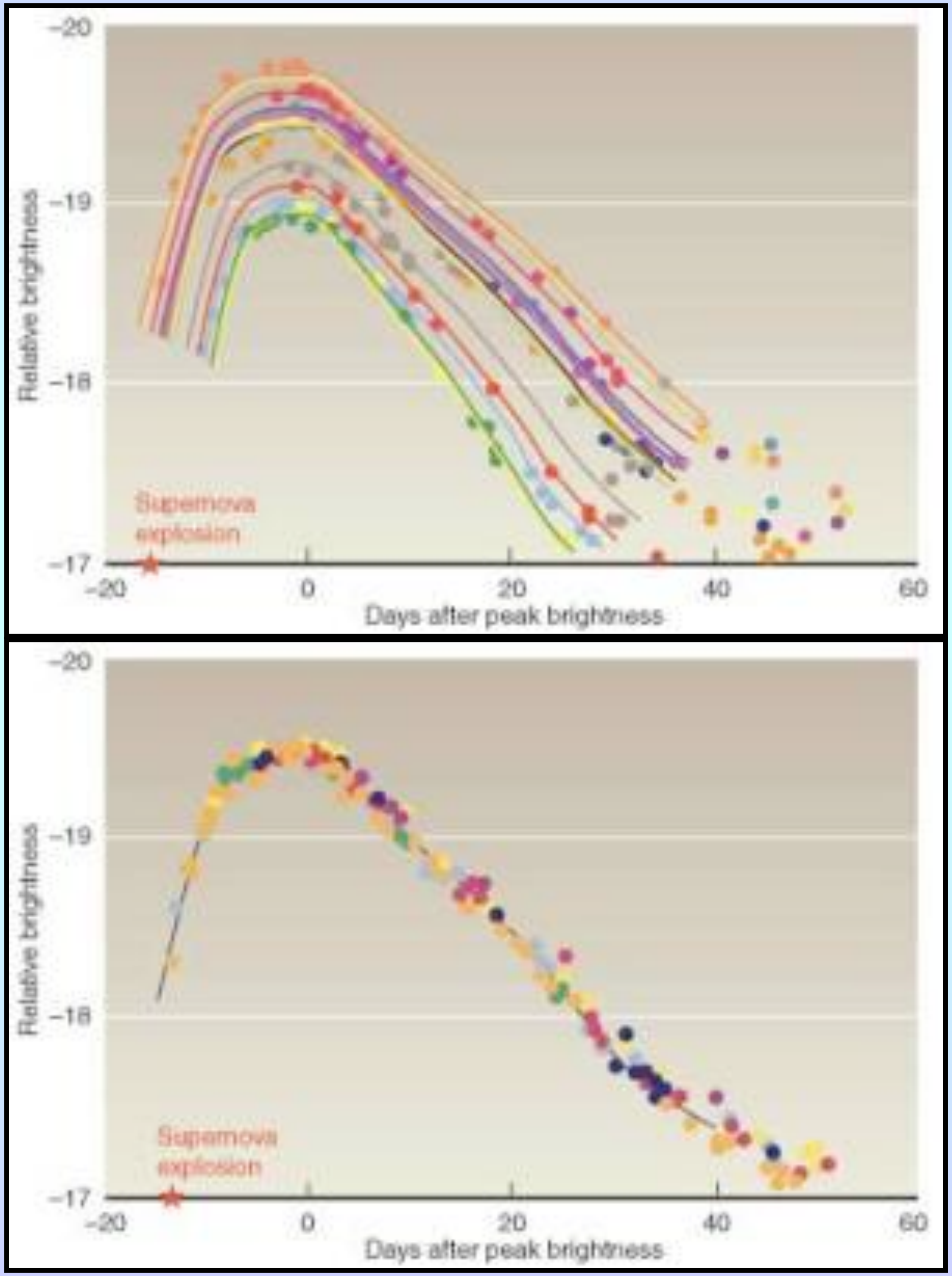
Like the Tully-Fisher method, SN Ia distances have a rather checkered history. Before 1985, there were no SN Ia and Ib; just SN I. Then, all SN Ia were thought to have the same absolute magnitude at maximum.

The break-through came in 1993, when Mark Phillips used a subset of nearby supernovae with well-determined distances to show a relationship between the maximum brightness of a SN Ia and the amount it faded in the first 15 days after maximum.



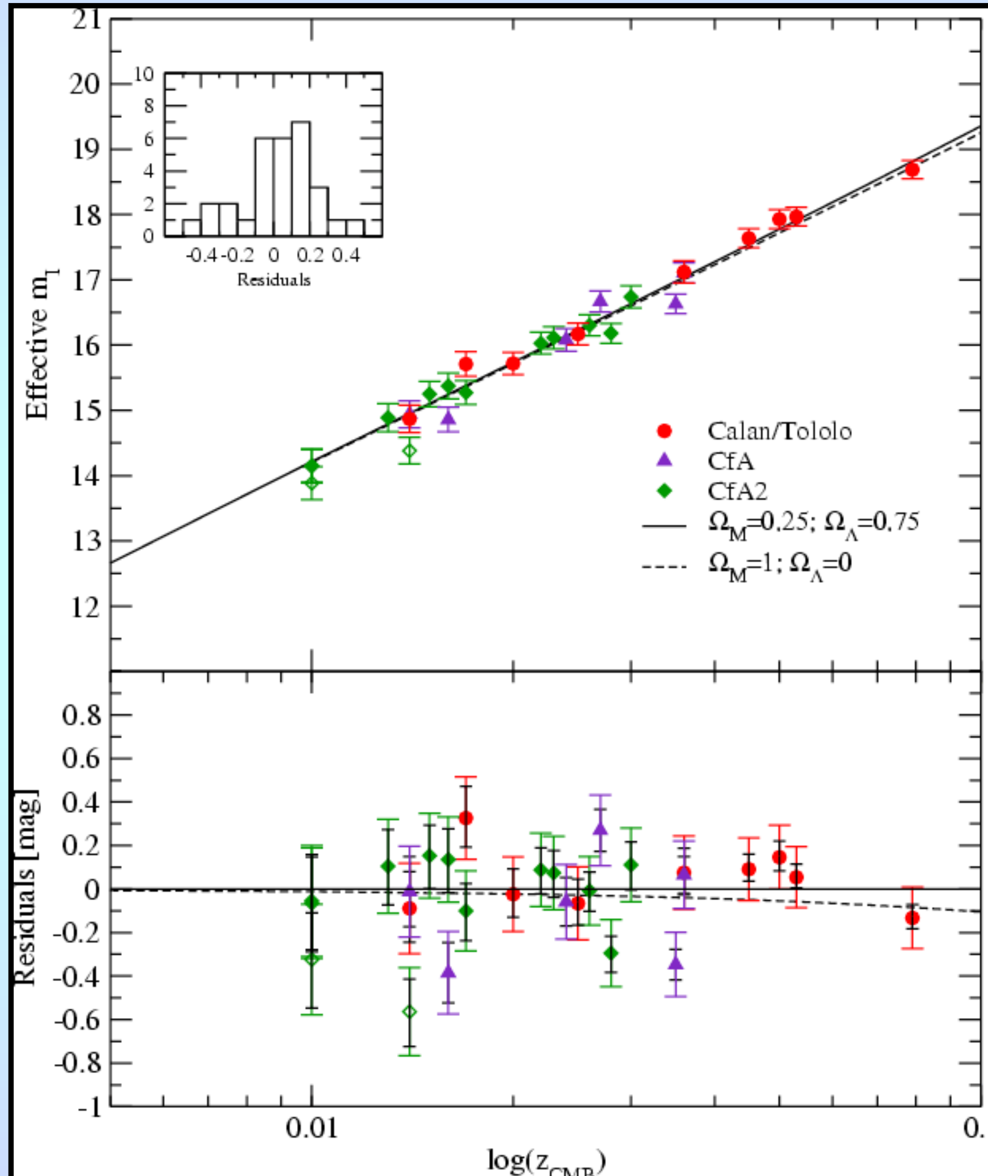
SN Ia Maximum Magnitude – Rate of Decline

The brighter the supernova at maximum, the longer it takes to fade. While a simple Δm_{15} does work, Adam Riess et al. (1996) improved on the relationship by parameterizing the light curves via their shape.



SN Ia Maximum Magnitude – Rate of Decline

The brighter the supernova at maximum, the longer it takes to fade. While a simple Δm_{15} does work, Adam Riess et al. (1996) improved on the relationship by parameterizing the light curves via their shape.



Aside: Luminosity Functions

A luminosity function describes the number of objects between magnitudes m and $m + \Delta m$. Note that the observed luminosity function, $N(m)$, is always the convolution of the true function, $S(m)$, with a function, $G(x)$ representing observational errors, i.e.,

$$N = S \circ G \quad \Rightarrow \quad N(m) = \int_{-\infty}^{+\infty} S(m - x)\, G(x)\, dx$$

Consequently, a power-law luminosity function will always appear flatter than the true function, and features in the function will be measured to be shallower. By expanding the convolution equation as a Taylor series, Eddington (1913) worked out a first-order correction for this effect, showing that for Gaussian errors,

$$S(m) = N(m) - \frac{\sigma^2}{2!} N''(m)$$

Aside: Luminosity Functions

Alternatively, one can consider an observed luminosity function as one manifestation of a probability distribution, $P(m)$. Suppose a set of observations extend down to some faint limit m_{lim}, then the probability of observing an object brighter than this limit is

$$\int_{m_{\lim}}^{-\infty} P(m)\,dm = 1$$

The probability of observing an object between m_1 and m_2 is

$$P(m_1 < m < m_2) = \frac{\int_{m_2}^{m_1} P(m)dm}{\int_{m_{\lim}}^{-\infty} P(m)dm}$$

And the probability of a particular set of magnitudes is

$$P_{tot} = \prod_i P(m_i)$$

Aside: Luminosity Functions

Using this probabilistic formulism, one can ask, "how much more/less likely is luminosity function X than luminosity function Y?". By trying all forms of the luminosity function, one can then find the most-likely one. Note:

- In this formulation, $P(m)$ represents the true luminosity function convolved with the photometric error function and including any selection effects.

- Often times, the multiplication of many probabilities together will create numerical difficulties for a computer. Consequently, one usually works with log probabilities, and then adds them.

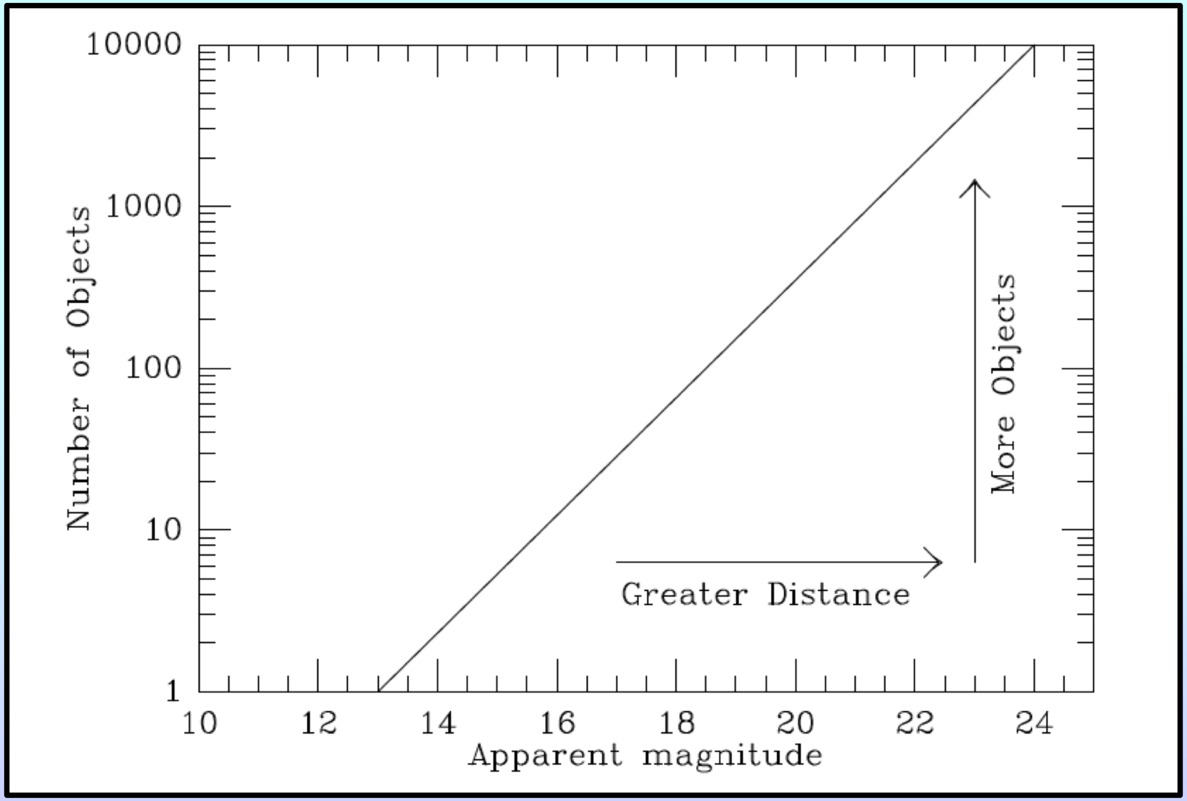
- A maximum-likelihood analysis can only find the best-fit among all the possibilities that are tested. It does not test for whether the "best fit" is any good.

Aside: Power Law Luminosity Functions

Power law luminosity functions, i.e., those of the form

$$N \propto L^{\alpha} \quad \Rightarrow \quad \log N = \alpha \log L + \beta$$

contain no distance information. The flux distribution for objects in a nearby low-luminosity galaxy will appear identical to that in a distant, high-luminosity galaxy. In the more distant galaxy, only the brightest objects will be visible, but they'll be more of them.

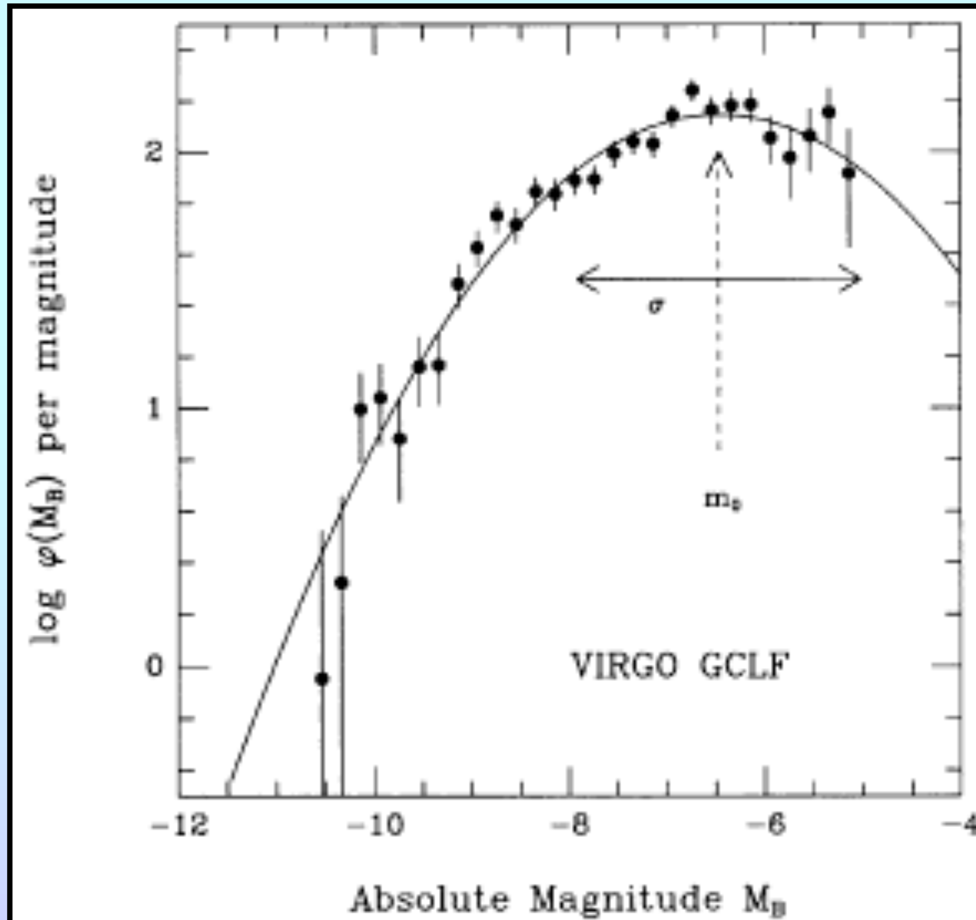


The Globular Cluster Luminosity Function

At a distance of > 1 Mpc, globular clusters look like a faint stars. But some galaxies have a lot of them, making them easy to count, even with background contamination. (This is especially true in elliptical and S0 systems.) In most galaxies, the GCLF looks like a log Gaussian, *i.e.*,

$$\log N \propto \exp\left\{-\frac{\left(M - M_0\right)^2}{2\sigma^2}\right\}$$

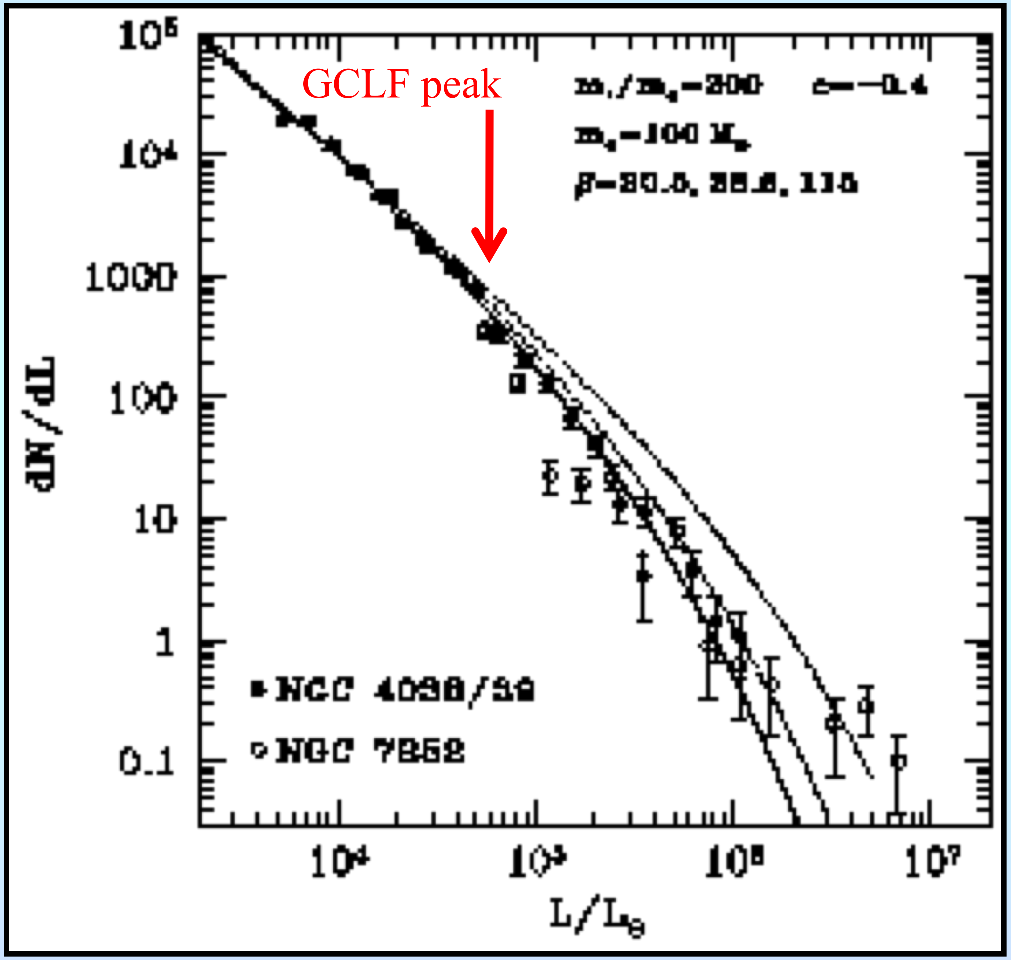
with $\sigma \sim 1.2$ and $M_0 = -7.3$ in V. The maximum observed peak, m_0 can then be compared with the observed peak to derive the distance.



The Globular Cluster Luminosity Function

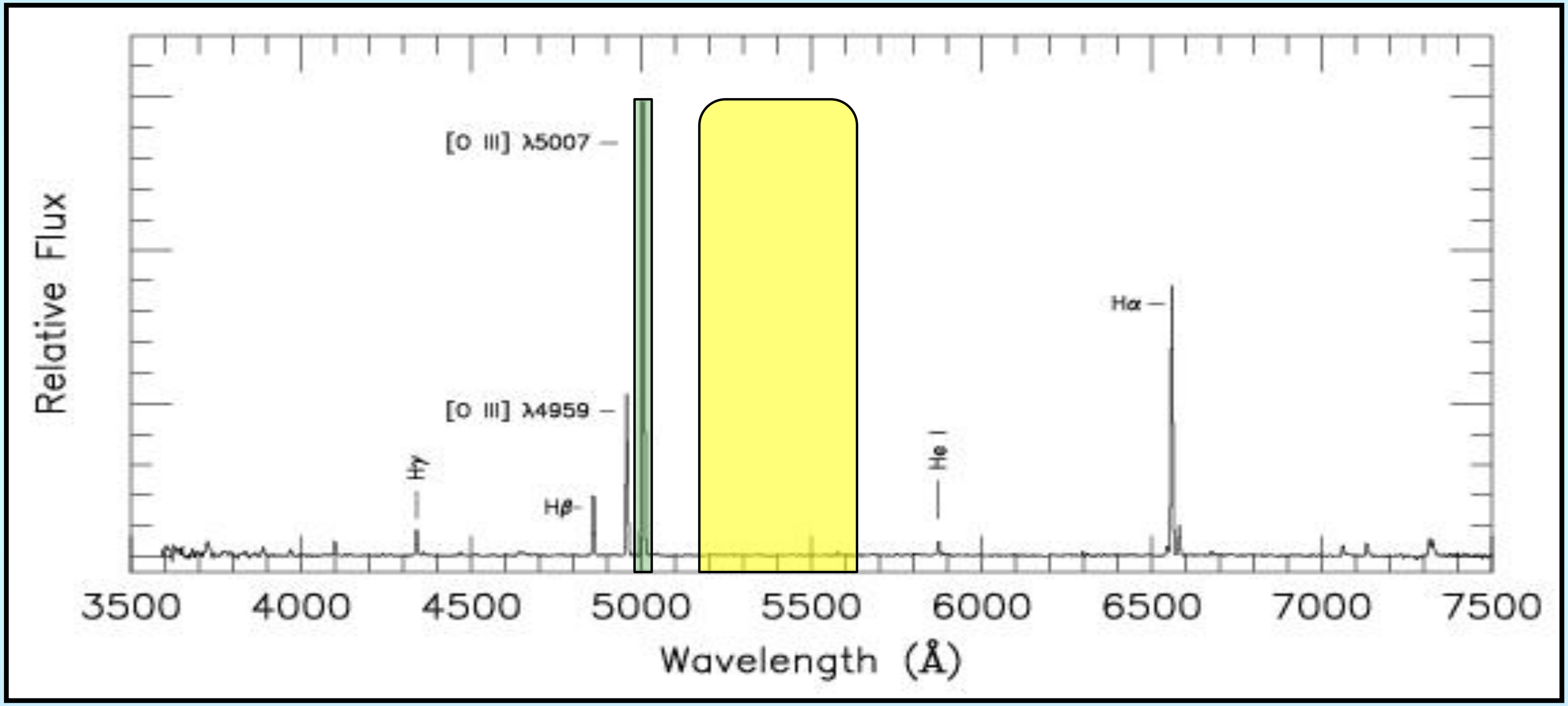
Note: globular clusters were almost certainly not born with their luminosity function.

In the Antennae galaxy and the LMC, star clusters are being born with a power-law luminosity function. Over time, this function likely evolves into a log-normal distribution, due to stellar evaporation and tidal stripping.



The Planetary Nebula Luminosity Function

Planetary nebulae, when viewed in the light of [O III] λ5007, follow a luminosity function that is (virtually) the same in all types of galaxies.



These objects are easily identified by taking pictures through narrow-band filters.

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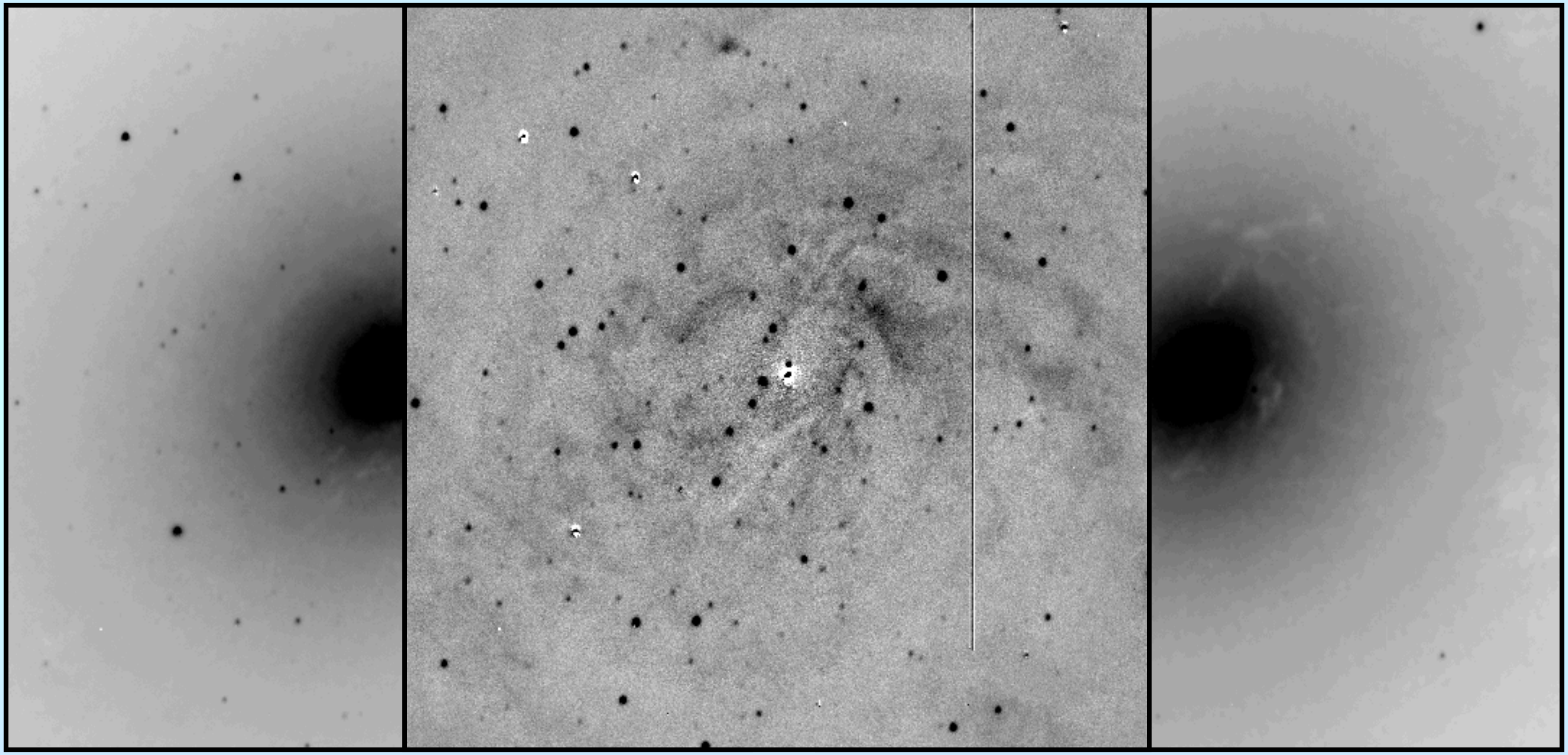
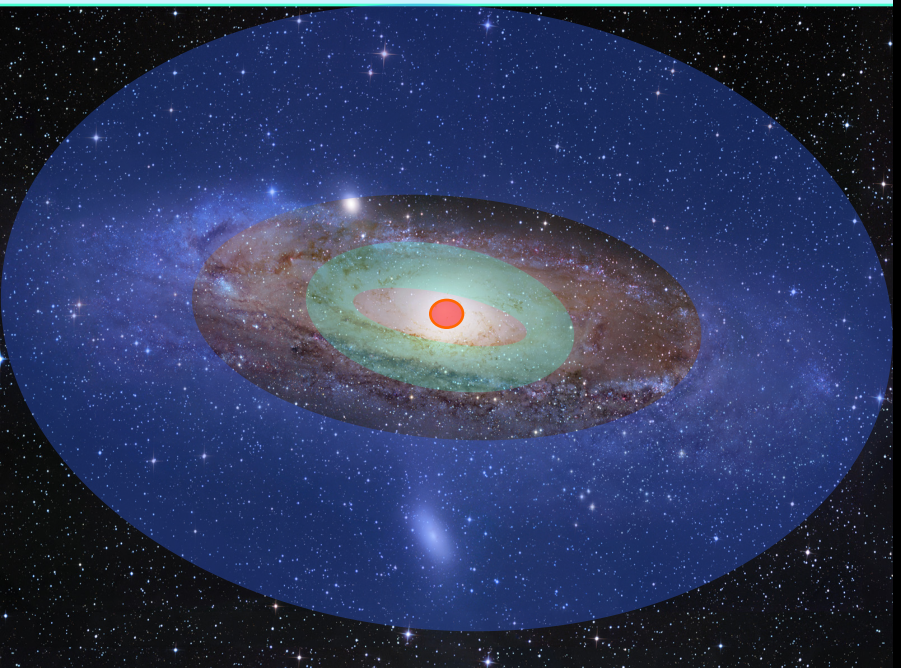
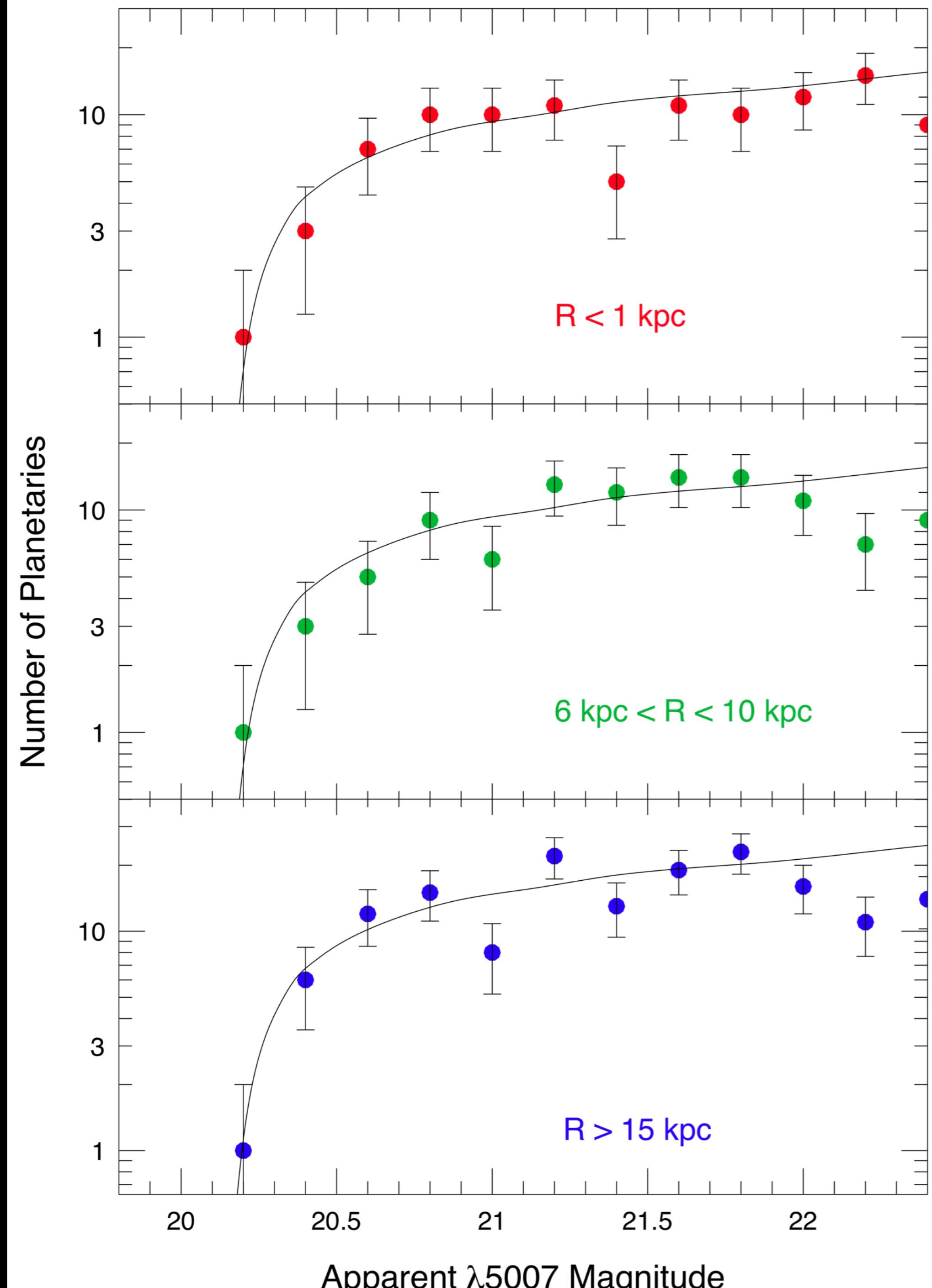


These objects are easily identified by taking pictures through narrow-band filters.

The PNLF of an old stellar population is the same as that produced by star-forming systems.



(Simple stellar evolution theory says this is impossible, so the PN probably come from binaries.)



PNLF

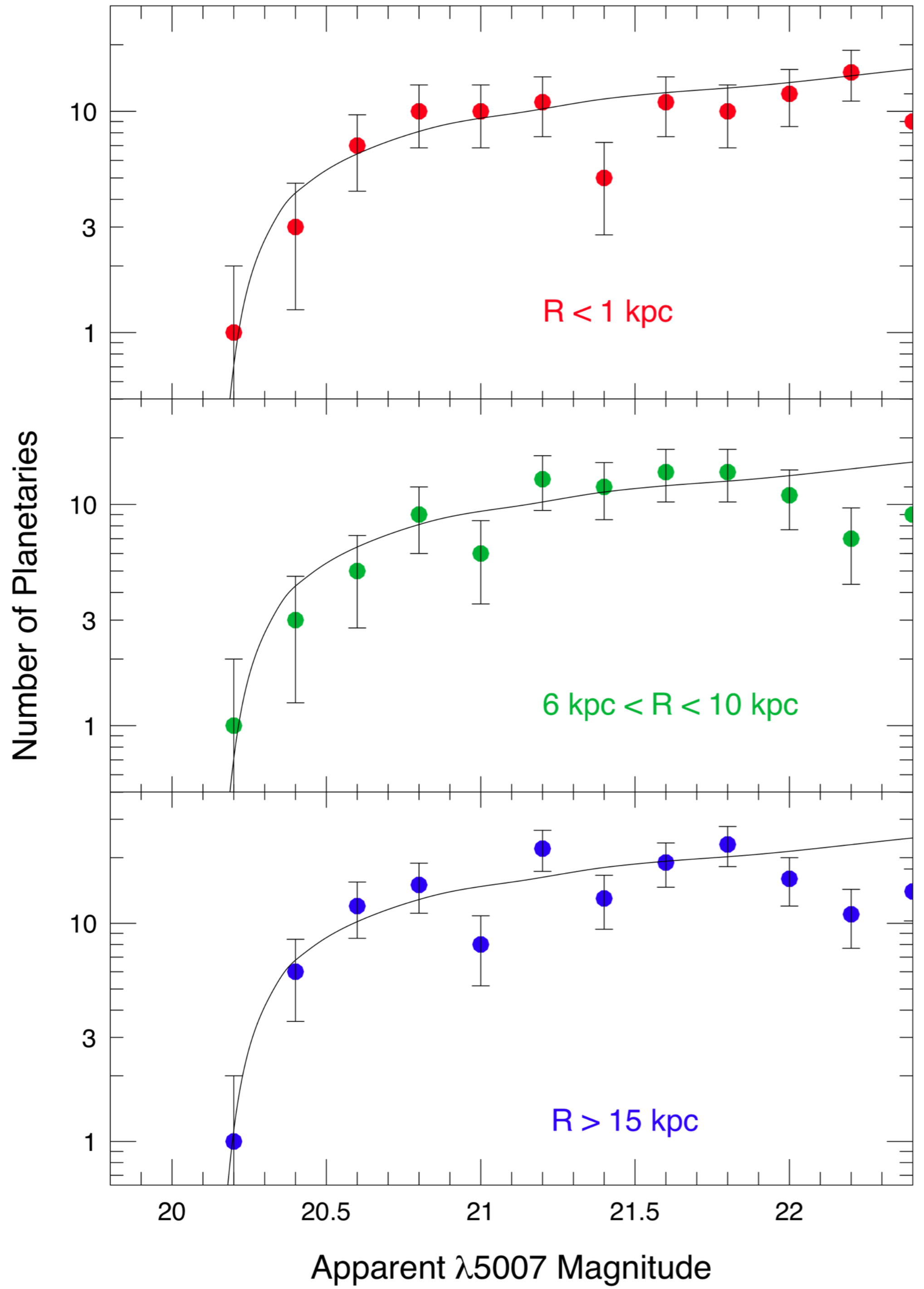
A simple analytical representation of the PNLF is

$$N(M) \propto e^{0.307M}\left\{1 - e^{3(M^* - M)}\right\}$$

where $M^* = -4.54$, and the $\lambda 5007$ magnitude systems is defined through

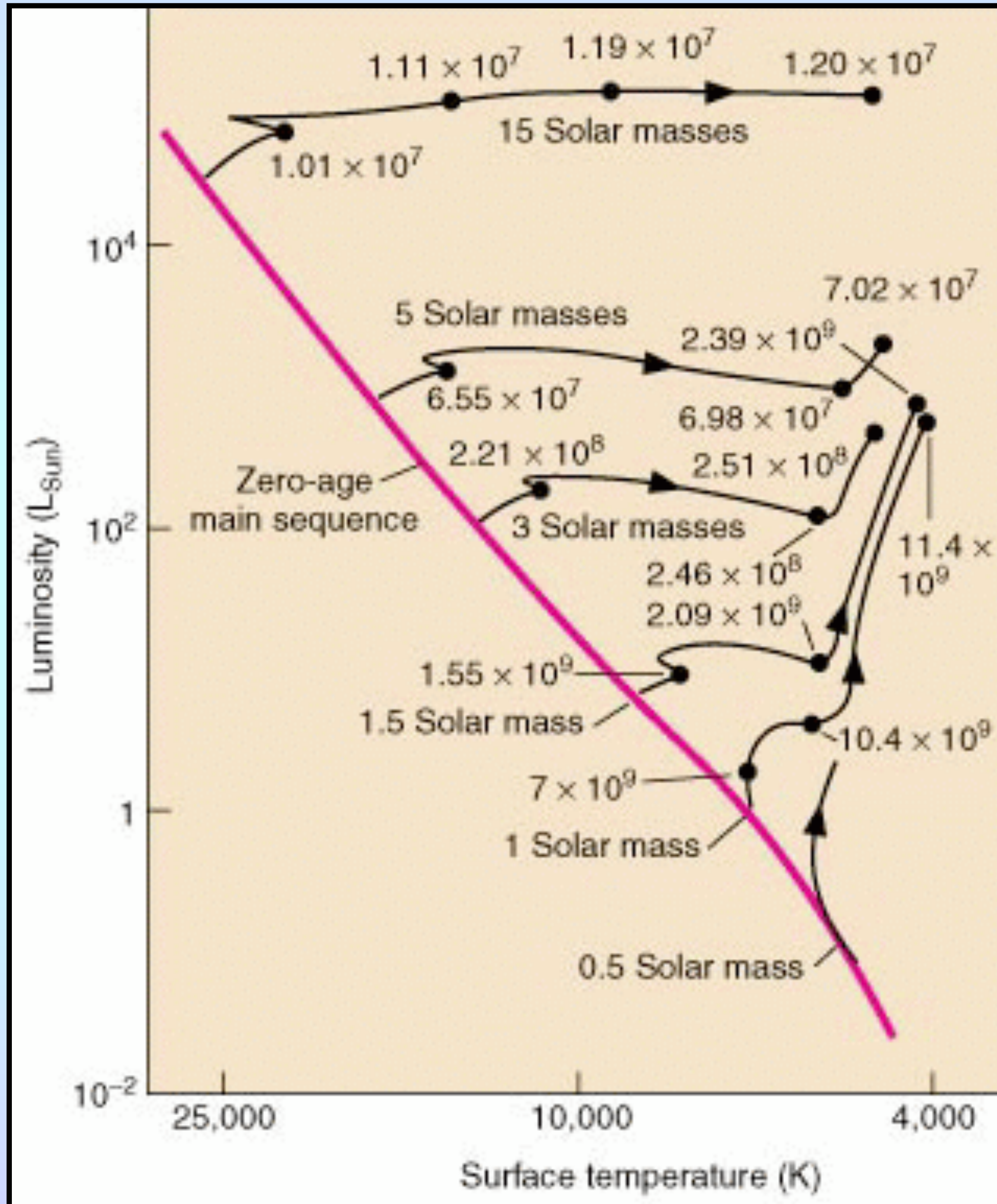
$$m_{5007} = -2.5 \log F_{5007} - 13.74$$

with F_{5007} in in ergs-cm^{-2}-s^{-1}.



The Tip of the Red Giant Branch

Because of electron degeneracy pressure, all red giant stars with masses $M < 3\ M_\odot$ reach about same maximum magnitude. So for old populations, the TRGB is independent of age.



The Tip of the Red Giant Branch

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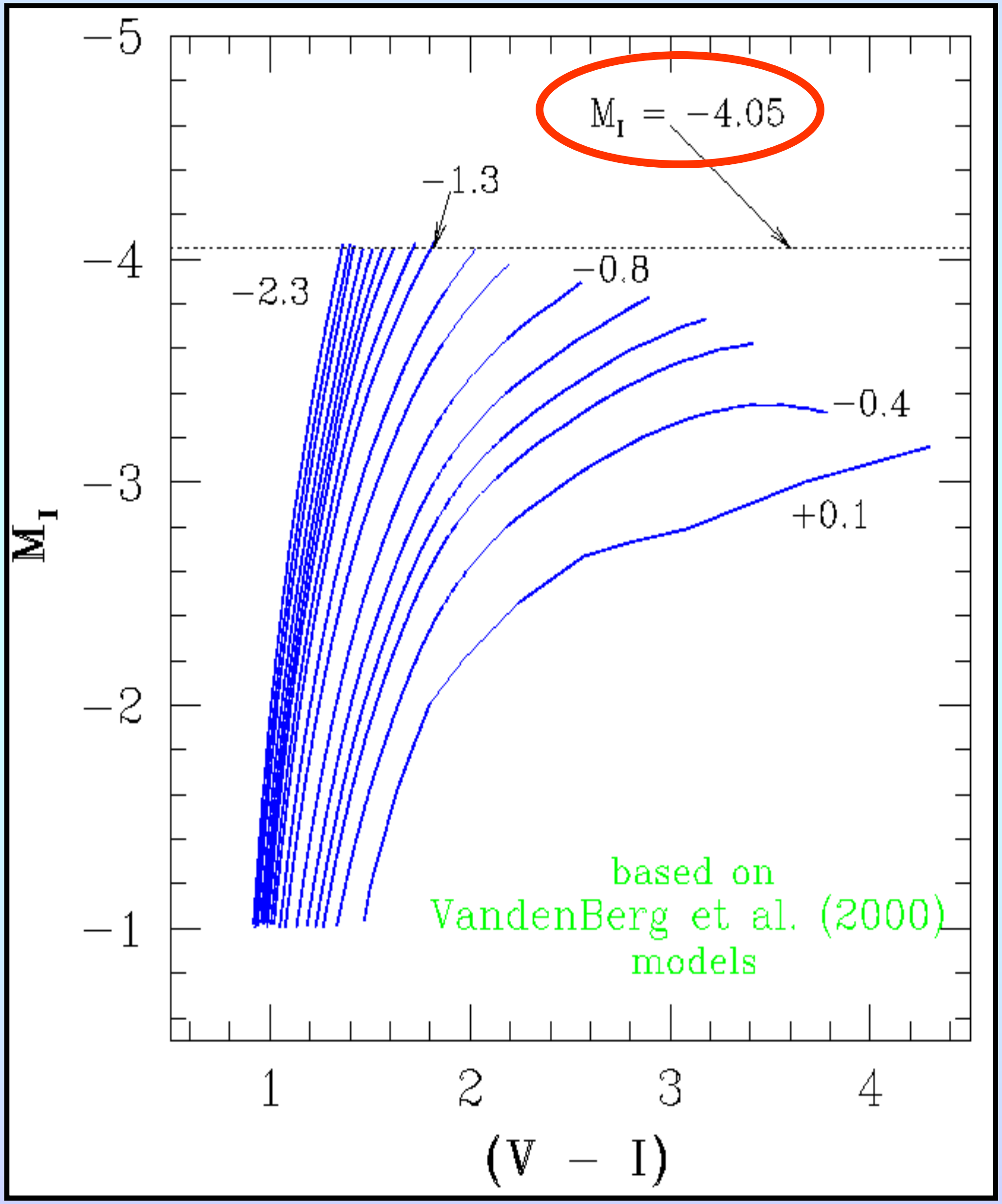
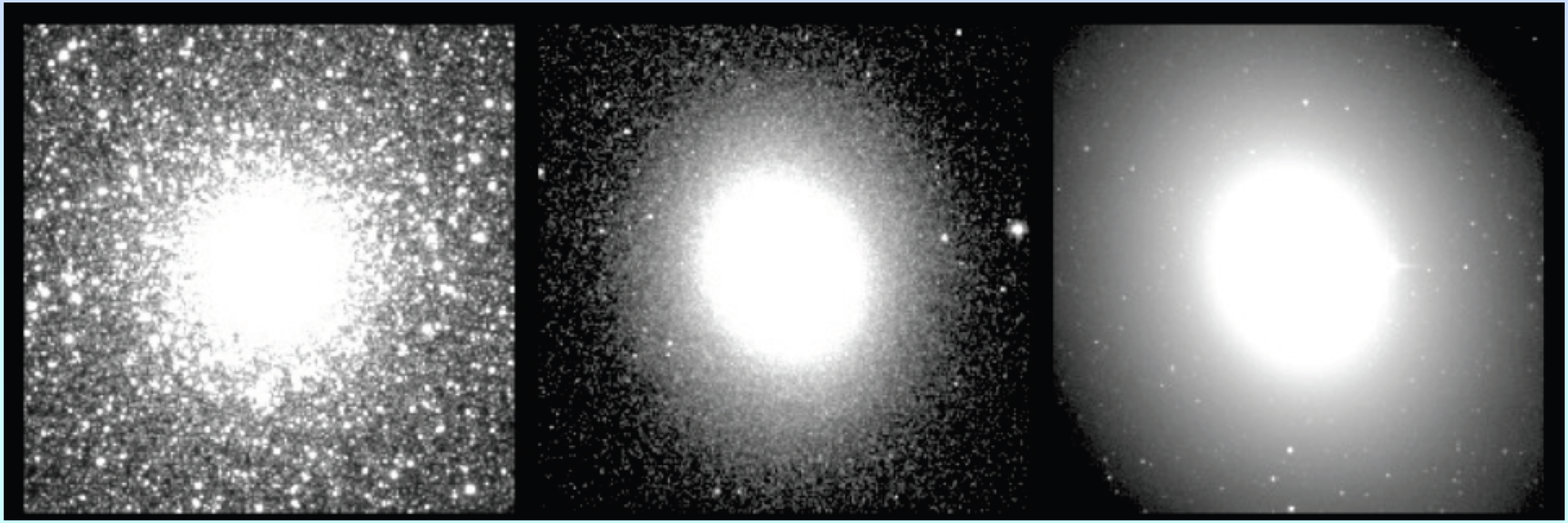
The maximum RGB magnitude also doesn't depend on metal abundance (at least for abundances less than about half solar)

Thus the RGB maximum magnitude is a standard candle.

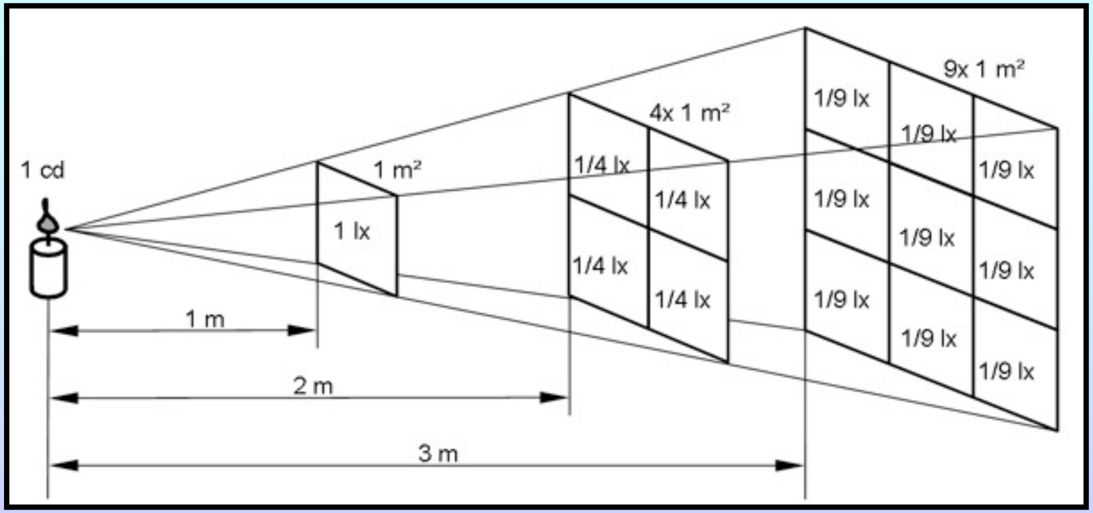


Surface Brightness Fluctuations



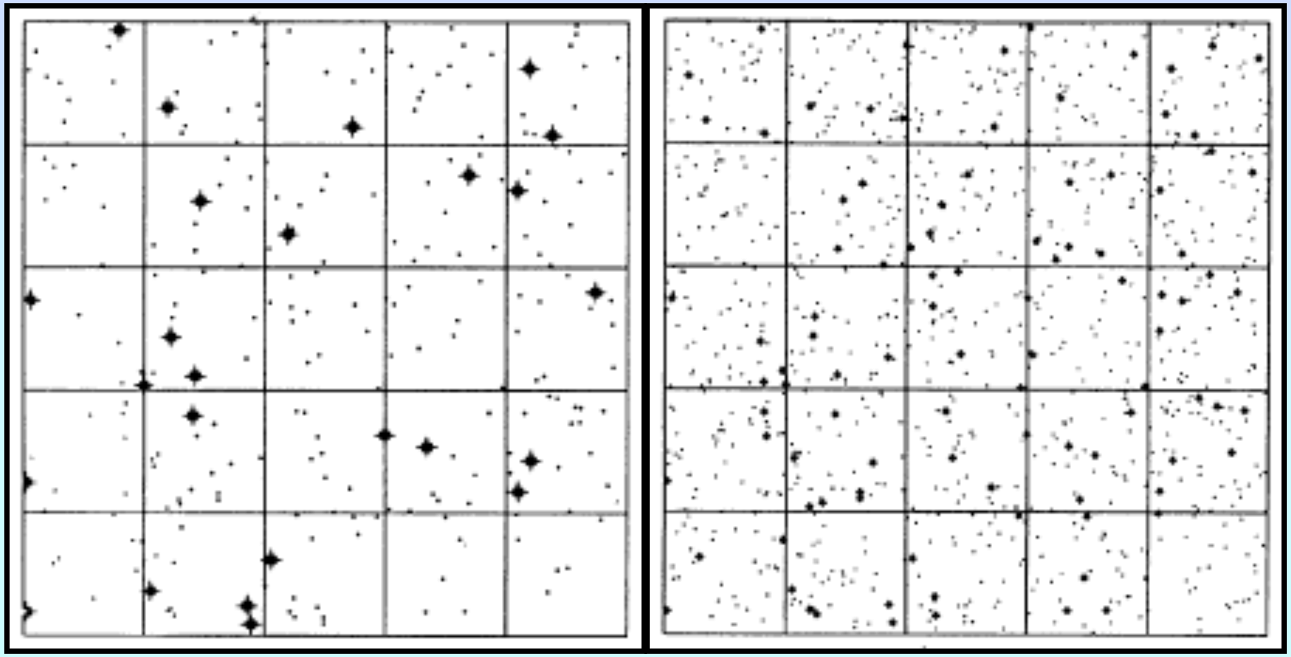
Which system is closer? How can you tell?

As the distance increases, the flux per star decreases by $1/r^2$, but the area increases by r^2. So surface brightness is independent of distance. But…



Surface Brightness Fluctuations

If one just counts the bright stars, the (percentage) pixel-to-pixel fluctuations of the nearer galaxy is greater.



The mean flux in each pixel is: $\langle F \rangle \propto \left(nr^2\right)\left(\overline{L}/r^2\right) \propto n\overline{L}$

The standard deviation between pixels is: $\sigma_{\langle F \rangle} \propto \left(nr^2\right)^{1/2}\left(\overline{L}/r^2\right)$

So the variance of the pixels is: $\sigma^2_{\langle F \rangle} \propto \left(nr^2\right)\left(\overline{L}/r^2\right)^2 \propto n\overline{L}^2/r^2$

And the percentage fluctuation is: $\dfrac{\sigma^2_{\langle F \rangle}}{\langle F \rangle} = \dfrac{n\overline{L}^2/r^2}{n\overline{L}} = \dfrac{n\overline{L}^2}{n\overline{L}}\dfrac{1}{r^2}$

The distance can therefore be obtained from a measurement of “noise”.

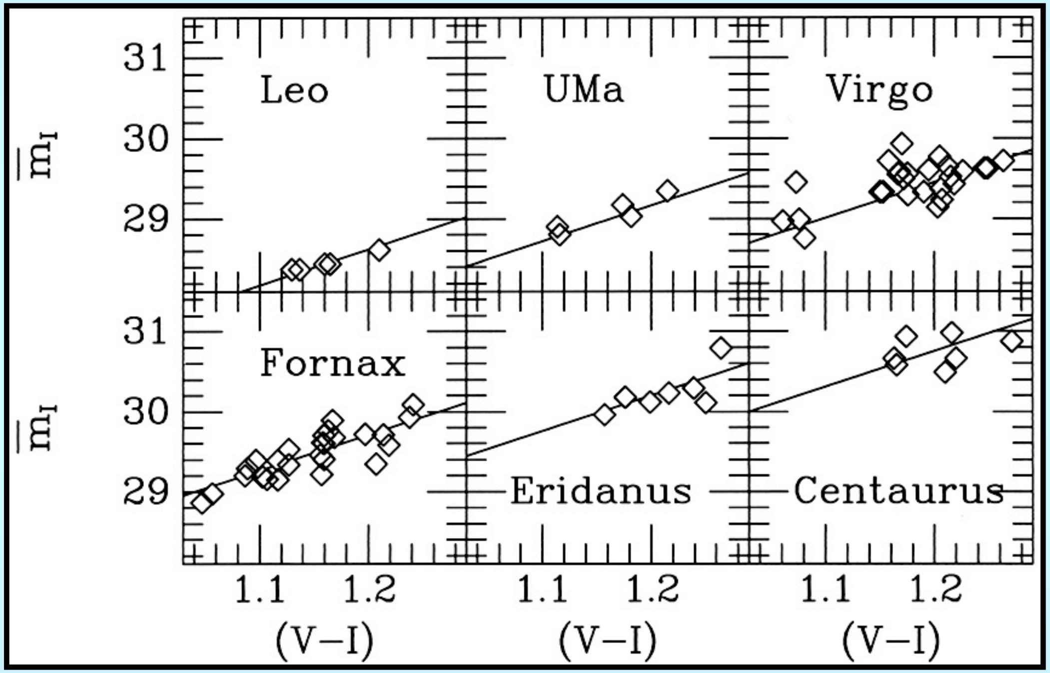
Surface Brightness Fluctuations

The "standard candle" for the SBF method is the fluctuation magnitude:

$$\overline{L} = \frac{\sum N_i L_i^2}{\sum N_i L_i}$$

This value depends on the color of the stellar population through age/metallicity (with redder systems having fainter fluctuation magnitudes).

$$\overline{M}_I = -1.74 + 4.5\left\{(V-I)_0 - 1.15\right\}$$



Note: since the method relies on measuring statistical noise, any other noise source (say, from dust or clumpy stellar distributions) will throw off the calculation. Thus, the method is restricted to smooth stellar populations, such as those found in elliptical galaxies.